

A DOUBLE INTEGRAL

OVIDIU FURDUI

Let $k \geq 2$ be a natural number and let $a_i, i = 1, \dots, k$ be natural numbers. Evaluate

$$\int_0^\infty \int_0^\infty \frac{(e^{-a_1x} - e^{-a_1y}) \dots (e^{-a_kx} - e^{-a_ky})}{(x-y)^2} dx dy.$$

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A SEQUENCE OF RATIONAL NUMBERS

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Find

$$\lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{n}{\frac{1}{2} + \frac{2}{3} + \cdots + \frac{n}{n+1}} \right)^n.$$

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A SIMPLE LIMIT

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Find the limit

$$\lim_{n \rightarrow \infty} \int_0^1 \frac{dx}{1 + \cos^2 x \cos^2(2x) \cdots \cos^2(nx)}.$$

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THE LIMIT OF AN INTEGRAL SEQUENCE

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Let a and b be real numbers and let c be a positive number. Find the limit

$$\lim_{n \rightarrow \infty} \int_a^b \frac{dx}{c + \sin^2 x \cdot \sin^2(x+1) \cdots \sin^2(x+n)}.$$

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A QUICKIE

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Prove that

$$\sum_{n=1}^{\infty} \frac{1}{n} \left(\sum_{k=1}^n \frac{x^k}{k} - \ln \frac{1}{1-x} \right) = -\frac{\ln^2(1-x)}{2}, \quad -1 \leq x < 1.$$

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PROPOSED PROBLEM TO CRUX MATHEMATICORUM WITH
MATHEMATICAL MAYHEM

OVIDIU FURDUI

Find the sum

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n} \left(1 - \frac{1}{2} + \frac{1}{3} - \cdots + \frac{(-1)^{n-1}}{n} \right).$$

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PROPOSED PROBLEM TO THE HARVARD COLLEGE MATHEMATICS
REVIEW
A TRICKY LIMIT

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Let $a, b \geq 0$ be two nonnegative numbers. Find the limit

$$\lim_{n \rightarrow \infty} \sum_{k=1}^n \frac{1}{n+k+b+\sqrt{n^2+kn+a}}.$$

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PROPOSED PROBLEM TO SCHOOL SCIENCE AND MATHEMATICS

OVIDIU FURDUI

Find the sum

$$\sum_{k=2}^{\infty} (-1)^k \ln \left(1 - \frac{1}{k^2} \right).$$

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THE LIMIT OF A PRODUCT

OVIDIU FURDUI

Let α be a real number and let $p > 1$. Find:

$$\lim_{n \rightarrow \infty} \prod_{k=1}^n \frac{n^p + (\alpha - 1)k^{p-1}}{n^p - k^{p-1}}.$$

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**PROPOSED PROBLEM TO REVISTA ESCOLAR DE LA OLIMPIADA
IBEROAMERICANA DE MATEMATICA**

OVIDIU FURDUI

Let n be a positive integer. Evaluate

$$\int_{\frac{1}{n}}^1 \cos(\pi \{nx\}) dx,$$

where $\{a\} = a - \lfloor a \rfloor$ is the *fractional part* of a .

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PROPOSED PROBLEM TO THE COLLEGE MATHEMATICS JOURNAL
AN IMPROPER EXPONENTIAL INTEGRAL

OVIDIU FURDUI

Let n be a fixed nonnegative integer. Evaluate

$$\int_0^{\infty} \frac{x^n}{e^x + 1 + x + \frac{x^2}{2!} + \cdots + \frac{x^n}{n!}} dx.$$

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PROPOSED PROBLEM TO CRUX MATHEMATICORUM WITH
MATHEMATICAL MAYHEM
A QUARTIC FRACTIONAL INTEGRAL

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Evaluate

$$\int_0^1 \left\{ \frac{1}{x} \right\}^4 dx,$$

where $\{x\} = x - \lfloor x \rfloor$ is the *fractional part* of x .

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PROPOSED PROBLEM TO THE MATHEMATICAL GAZETTE

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Let $n \geq 1$ and $m \geq 1$ be natural numbers. Evaluate

$$I = \int_0^{\infty} \int_0^{\infty} \frac{(e^{-nx} - e^{-ny})(e^{-mx} - e^{-my})}{(x - y)^2} dx dy.$$

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A GEOMETRY PROBLEM

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Let $ABCD$ be a convex quadrilateral and let P be a point in the interior of $ABCD$ such that $PA = \frac{AB\sqrt{2}}{2}$, $PB = \frac{BC\sqrt{2}}{2}$, $PC = \frac{CD\sqrt{2}}{2}$ and $PD = \frac{DA\sqrt{2}}{2}$. Prove or disprove that $ABCD$ is a square.

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11234. Let $a_1, a_2, \dots, a_n$ and $b_1, \dots, b_{n-1}$ be real numbers with $a_1 < b_1 < a_2 < \dots < a_{n-1} < b_{n-1} < a_n$, and let h be an integrable function from R to R . Show that

$$\int_{-\infty}^{\infty} h \left(\frac{(x - a_1) \cdots (x - a_n)}{(x - b_1) \cdots (x - b_{n-1})} \right) dx = \int_{-\infty}^{\infty} h(x) dx.$$