

CD1: THỰC HIỆN PHÉP TÍNH
DẠNG 6: TÍNH TÍCH

A. Trắc nghiệm

Câu 1. (HSG 7 huyện Thanh Thủy, tỉnh Phú Thọ, năm 2021 - 2022)

$$A = \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{2022 \cdot 2023}$$

Rút gọn biểu thức: ta được

A. $A = \frac{3}{2}$ B. $A = \frac{2021}{2022}$ C. $A = \frac{2023}{2}$ D. $A = \frac{2}{2021}$

Lời giải

Chọn C

$$A = \frac{1}{2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{2022 \cdot 2023} = \frac{3}{2} \times \frac{4}{3} \times \frac{5}{4} \times \frac{2023}{2022} = \frac{2023}{2}$$

Vậy chọn đáp án C.

Câu 2. (HSG 7 Đề khảo sát lần 3, 2022 - 2023)

$$B = \left(1 - \frac{1}{1+2}\right) \cdot \left(1 - \frac{1}{1+2+3}\right) \cdots \left(1 - \frac{1}{1+2+\cdots+2021}\right)$$

Tính giá trị biểu thức:

A. $\frac{671}{2021}$ B. $\frac{2023}{6063}$ C. $\frac{2019}{6063}$ D. $\frac{2020}{6063}$

Lời giải

Chọn D

$$B = \left(1 - \frac{1}{1+2}\right) \cdot \left(1 - \frac{1}{1+2+3}\right) \cdots \left(1 - \frac{1}{1+2+\cdots+2021}\right)$$

$$B = \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdots \frac{2022}{2023} = \frac{2 \cdot 3 \cdot 4 \cdots 2022}{3 \cdot 4 \cdot 5 \cdots 2023} = \frac{2}{2023}$$

$$= \frac{2}{3} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdots \frac{2022}{2023} = \frac{2 \cdot 3 \cdot 4 \cdots 2022}{3 \cdot 4 \cdot 5 \cdots 2023} = \frac{2}{2023}$$

$$= \frac{1 \cdot 4}{2 \cdot 3} \cdot \frac{2 \cdot 5}{3 \cdot 4} \cdot \frac{3 \cdot 6}{4 \cdot 5} \cdots \frac{2020 \cdot 2023}{2021 \cdot 2022} = \frac{2023}{6063}$$

Câu 3. (HSG 7 Đề khảo sát lần 3, 2022 - 2023)

$$\left(\frac{3}{4} - 81\right) \cdot \left(\frac{3^2}{5} - 81\right) \cdot \left(\frac{3^3}{6} - 81\right) \cdots \left(\frac{3^{2018}}{2021} - 81\right)$$

Tính giá trị biểu thức:

A. 0 B. 1 C. 2021 D. 3^{2021}

Lời giải

Chọn A

$$\frac{3^6}{9} - 81 \frac{3^5}{5} - 81 \frac{3^4}{6} - 81 \frac{3^3}{7} \dots - 81 \frac{3^{2018}}{2021} - 81 \frac{3^{2019}}{2022}$$

$$\frac{3^6}{9} - 81 = 0$$

Ta thấy: trong dãy số trên có

$$\frac{3^6}{9} - 81 \frac{3^5}{5} - 81 \frac{3^4}{6} - 81 \frac{3^3}{7} \dots - 81 \frac{3^{2018}}{2021} - 81 \frac{3^{2019}}{2022} = 0$$

Do đó

B. Tự luận

Câu 1. (HSG 7 TP Thanh Hoá năm 2022-2023)

$$A = \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{2019} - \frac{1}{2020} + \frac{1}{2021} - \frac{1}{2022}$$

Cho

$$B = \frac{1}{2} + \frac{1}{2019} - \frac{1}{3} + \frac{1}{2020} - \frac{1}{4} + \frac{1}{2021} - \frac{1}{5} + \dots - \frac{1}{2019} + \frac{1}{2020} - \frac{1}{2021} + \frac{1}{2022}$$

Tính giá trị biểu thức $M = A.B$.

Lời giải

$$\begin{aligned} A &= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{2019} - \frac{1}{2020} + \frac{1}{2021} - \frac{1}{2022} \\ &= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{2019} - \frac{1}{2020} + \frac{1}{2021} - \frac{1}{2022} \quad (2020 \text{ thừa số}) \\ &= \frac{1}{2021} \end{aligned}$$

$$\begin{aligned} B &= \frac{1}{2} + \frac{1}{2019} - \frac{1}{3} + \frac{1}{2020} - \frac{1}{4} + \frac{1}{2021} - \frac{1}{5} + \dots - \frac{1}{2019} + \frac{1}{2020} - \frac{1}{2021} + \frac{1}{2022} \\ &= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \dots - \frac{1}{2019} + \frac{1}{2020} - \frac{1}{2021} + \frac{1}{2022} \quad (2019 \text{ thừa số}) \\ &= -\frac{2021}{2} \end{aligned}$$

$$M = A.B = \frac{1}{2021} \cdot -\frac{2021}{2} = -\frac{1}{2}$$

Suy ra:

Câu 2. (HSG 7 Thọ Xuân Thanh Hoá năm 2022-2023)

$$B = \frac{1}{2} - \frac{1}{2022} + \frac{1}{3} - \frac{1}{2023} + \frac{1}{4} - \frac{1}{2024} + \dots - \frac{1}{2022} + \frac{1}{2023} - \frac{1}{2024} + \frac{1}{2025}$$

Tính giá trị biểu thức:

Lời giải

$$\begin{aligned} B &= \frac{1}{2} - \frac{1}{2022} + \frac{1}{3} - \frac{1}{2023} + \frac{1}{4} - \frac{1}{2024} + \dots - \frac{1}{2022} + \frac{1}{2023} - \frac{1}{2024} + \frac{1}{2025} \\ &= \frac{1}{2} - \frac{1}{2022} + \frac{1}{3} - \frac{1}{2023} + \frac{1}{4} - \frac{1}{2024} + \dots - \frac{1}{2022} + \frac{1}{2023} - \frac{1}{2024} + \frac{1}{2025} = \frac{1}{2023} \end{aligned}$$

Câu 3. (HSG 7 Hâu Lộc năm 2022-2023)

$$P = \frac{1}{1} + \frac{7}{9} + \frac{7}{20} + \frac{7}{33} + \frac{7}{2900}$$

Tính tích

Lời giải

$$P = \frac{1}{1} + \frac{7}{9} + \frac{7}{20} + \frac{7}{33} + \frac{7}{2900}$$

$$= \frac{2.8.3.9.4.10...51.57}{1.9.2.10.3.11...50.58} = \frac{2.3.4...51}{1.2.3...50} \cdot \frac{8.9.10...57}{9.10.11...58} = \frac{51.8}{58} = \frac{204}{29}$$

$$P = \frac{204}{29}$$

Vậy

Câu 4. (HSG 7 Lập Thạch-Thanh Hoá năm 2022-2023)

$$B = \frac{1}{1} - \frac{1}{3} - \frac{1}{6} - \frac{1}{10} - \frac{1}{15} - \frac{1}{210}$$

Tính bằng cách hợp lí:

Lời giải

$$B = \frac{1}{1} - \frac{1}{3} - \frac{1}{6} - \frac{1}{10} - \frac{1}{15} - \frac{1}{210}$$

$$= \frac{1}{1} - \frac{1}{3} - \frac{1}{6} - \frac{1}{10} - \frac{1}{15} - \frac{1}{210}$$

$$= \frac{(1.4).(2.5).(3.6).(4.7).....(19.22)}{(2.3).(3.4).(4.5).(5.6).....(20.21)} = \frac{(1.2.3....19).(4.5.6.7....22)}{(2.3.4....20).(3.4.5.6.....21)} = \frac{11}{30}$$

$$B = \frac{11}{30}$$

Vậy

Câu 5. (HSG 7 TX Hương Trà 2022 - 2023)

$$B = \frac{\left(\frac{1}{3} - \frac{3}{4}\right)^{2023}}{\frac{5}{12}}$$

Tính:

Lời giải

$$B = \frac{\left(\frac{1}{3} - \frac{3}{4}\right)^{2023}}{\frac{5}{12}} = \frac{\frac{8}{27} \times \frac{9}{16} \times (-1)}{\frac{4}{25} \times \frac{125}{1728}} = \frac{-1}{6} : \frac{5}{432} = \frac{1}{6} \times \frac{432}{5} = \frac{72}{5}$$

Câu 6. (HSG 7 huyện Yên Bình – Yên Bái 2022 - 2023)

$$C = \frac{1}{1} - \frac{1}{4} - \frac{1}{9} - \frac{1}{16} - \frac{1}{400}$$

Tính giá trị các biểu thức:

Lời giải

$$C = \frac{1}{4 \cdot 9} - \frac{1}{4 \cdot 16} - \frac{1}{9 \cdot 16} - \frac{1}{16 \cdot 25} + \frac{1}{400}$$

$$C = \frac{3}{4} \times \frac{8}{9} \times \frac{15}{16} \times \frac{399}{400} = \frac{1.3}{2.2} \times \frac{2.4}{3.3} \times \frac{3.5}{4.4} \times \frac{19.21}{20.20} = \frac{(1.2.3 \dots 19)(3.4.5 \dots 21)}{(2.3.4 \dots 20)(2.3.4.5 \dots 20)}$$

$$C = \frac{21}{20.2} = \frac{21}{40}$$

$$C = \frac{21}{40}$$

Vậy

Câu 7. (HSG 7 huyện Nông Cống 2022 - 2023)

Tính giá trị của biểu thức sau:

$$C = \frac{1}{54.9} + \frac{1}{9.14} + \frac{1}{14.19} + \dots + \frac{1}{44.49} - \frac{1-3-5-7-\dots-49}{90}$$

Lời giải

$$C = \frac{1}{54.9} + \frac{1}{9.14} + \frac{1}{14.19} + \dots + \frac{1}{44.49} - \frac{1-3-5-7-\dots-49}{90}$$

$$C = \frac{1}{54} - \frac{1}{9} + \frac{1}{9} - \frac{1}{14} + \frac{1}{14} - \frac{1}{19} + \dots + \frac{1}{44} - \frac{1-3-5-7-\dots-49}{90}$$

$$C = \frac{1}{54} - \frac{1-52.24:2}{49 \cdot 90} = \frac{9}{4.49} \times \frac{623}{90} = -\frac{9}{4.49} \times \frac{7.89}{90} = -\frac{89}{280}$$

$$C = -\frac{89}{280}$$

Vậy

Câu 8. (HSG 7)

$$A = \frac{2^2}{1.3} \cdot \frac{3^2}{2.4} \cdot \frac{4^2}{3.5} \dots \frac{20^2}{19.21}$$

Tính tích:

Lời giải

$$A = \frac{2.2}{1.3} \cdot \frac{3.3}{2.4} \cdot \frac{4.4}{3.5} \dots \frac{20.20}{19.21} = \frac{(2.3.4 \dots 20)(2.3.4 \dots 20)}{(1.2.3 \dots 19)(3.4.5 \dots 21)} = \frac{20.2}{21} = \frac{40}{21}$$

Ta có:

Câu 9. (HSG 7)

$$C = \frac{1}{1+2} - \frac{1}{1+2+3} - \frac{1}{1+2+3+4} + \dots - \frac{1}{1+2+3+\dots+2016}$$

Tính tích:

Lời giải

$$C = \frac{1}{1+2} - \frac{1}{1+2+3} - \frac{1}{1+2+3+4} + \dots - \frac{1}{1+2+3+\dots+2016}$$

$$= \frac{1}{(1+2) \cdot 2} - \frac{1}{(1+3) \cdot 3} - \frac{1}{(1+4) \cdot 4} + \dots - \frac{1}{(1+2016) \cdot 2016}$$

$$= \frac{2}{3} \cdot \frac{5}{6} \cdot \frac{9}{10} \cdots \frac{2017 \cdot 2016 - 2}{2016 \cdot 2017} = \frac{4}{6} \cdot \frac{10}{12} \cdot \frac{18}{20} \cdots \frac{2016 \cdot 2017 - 2}{2016 \cdot 2017}$$

$$= \frac{1 \cdot 4}{2 \cdot 3} \cdot \frac{2 \cdot 5}{3 \cdot 4} \cdot \frac{3 \cdot 6}{4 \cdot 5} \cdots \frac{2015 \cdot 2018}{2016 \cdot 2017} = \frac{1004}{3009}$$

Câu 10. (HSG 7)

Tính:

$$A = \frac{1}{2} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \frac{1}{11} - \frac{1}{17} + \frac{1}{23} - \frac{1}{31} + \frac{1}{41} - \frac{1}{53} + \frac{1}{71} - \frac{1}{97}$$

Lời giải

$$A = \frac{1}{2 \cdot 3} \cdot \frac{3}{2 \cdot 5} \cdot \frac{5}{2 \cdot 7} \cdots \frac{97}{2 \cdot 99} = \frac{(1 \cdot 3 \cdot 5 \cdots 97)}{2^{49} \cdot (3 \cdot 5 \cdot 7 \cdots 99)} = \frac{1}{2^{49} \cdot 99}$$

Ta có:

Câu 11. (HSG 7 Trường Trà My 2018 - 2019; trường Hiền Quan 2017 - 2018; trường Xuân Dương 2013 - 2014)

Tính

$$\frac{1}{4} - 81 \frac{1}{5^2} + 81 \frac{1}{5^3} - 81 \frac{1}{5^4} + \cdots + \frac{1}{2003} - 81 \frac{1}{2003^2}$$

Lời giải

$$\frac{1}{4} - 81 \frac{1}{5^2} + 81 \frac{1}{5^3} - 81 \frac{1}{5^4} + \cdots + \frac{1}{2003} - 81 \frac{1}{2003^2}$$

$$\frac{3^6}{9} - 81 = 0$$

Ta thấy: trong dãy số trên có

$$\frac{1}{4} - 81 \frac{1}{5^2} + 81 \frac{1}{5^3} - 81 \frac{1}{5^4} + \cdots + \frac{1}{2003} - 81 \frac{1}{2003^2} = 0$$

Do đó

Câu 12. (HSG 7 huyện Việt Yên 2017 - 2018)

$$B = \frac{1}{2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \frac{1}{7^2} - \frac{1}{8^2} + \frac{1}{9^2} - \frac{1}{10^2} + \frac{1}{11^2} - \frac{1}{12^2} + \frac{1}{13^2} - \frac{1}{14^2} + \frac{1}{15^2} - \frac{1}{16^2} + \frac{1}{17^2} - \frac{1}{18^2} + \frac{1}{19^2} - \frac{1}{20^2}$$

Tính:

Lời giải

$$B = \frac{1}{2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{6^2} + \frac{1}{7^2} - \frac{1}{8^2} + \frac{1}{9^2} - \frac{1}{10^2} + \frac{1}{11^2} - \frac{1}{12^2} + \frac{1}{13^2} - \frac{1}{14^2} + \frac{1}{15^2} - \frac{1}{16^2} + \frac{1}{17^2} - \frac{1}{18^2} + \frac{1}{19^2} - \frac{1}{20^2}$$

$$= \frac{1}{2} - \frac{3}{4} + \frac{4}{9} - \frac{5}{16} + \frac{6}{25} - \frac{7}{36} + \frac{8}{49} - \frac{9}{64} + \frac{10}{81} - \frac{11}{100} + \frac{12}{121} - \frac{13}{144} + \frac{14}{169} - \frac{15}{196} + \frac{16}{225} - \frac{17}{256} + \frac{18}{289} - \frac{19}{324} + \frac{20}{361} - \frac{21}{400}$$

$$= \frac{1}{2} - \frac{2}{3} + \frac{3}{4} - \frac{4}{5} + \frac{5}{6} - \frac{6}{7} + \frac{7}{8} - \frac{8}{9} + \frac{9}{10} - \frac{10}{11} + \frac{11}{12} - \frac{12}{13} + \frac{13}{14} - \frac{14}{15} + \frac{15}{16} - \frac{16}{17} + \frac{17}{18} - \frac{18}{19} + \frac{19}{20} - \frac{20}{21}$$

$$= \frac{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdots 100}{2 \cdot 3 \cdot 4 \cdot 5 \cdot 6 \cdots 101} = \frac{1}{101}$$

Câu 13. (HSG 7 huyện Kim Thành 2022 - 2023)

$$B = \frac{1}{1} + \frac{1}{1 \cdot 3} + \frac{1}{3 \cdot 5} + \frac{1}{5 \cdot 7} + \frac{1}{7 \cdot 9} + \frac{1}{9 \cdot 11} + \frac{1}{11 \cdot 13} + \frac{1}{13 \cdot 15} + \frac{1}{15 \cdot 17} + \frac{1}{17 \cdot 19} + \frac{1}{19 \cdot 21}$$

Tính hợp lý giá trị biểu thức

Lời giải

$$B = \frac{1}{1} + \frac{1}{1.3} + \frac{1}{1.3.5} + \frac{1}{1.3.5.7} + \frac{1}{1.3.5.7.9} + \frac{1}{19.21}$$

$$B = \frac{4}{1.3} \times \frac{9}{2.4} \times \frac{16}{3.5} \times \frac{1}{4} \times \frac{400}{19.21}$$

$$B = \frac{2.2}{1.3} \times \frac{3.3}{2.4} \times \frac{4.4}{3.5} \times \frac{1}{4} \times \frac{20.20}{19.21}$$

$$B = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{3}{2} \cdot \frac{3}{4} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{20}{19} \cdot \frac{20}{21}$$

$$B = \frac{2}{1} \cdot \frac{3}{2} \cdot \frac{4}{3} \cdot \frac{20}{19} \cdot \frac{3}{4} \cdot \frac{4}{5} \cdot \frac{20}{21}$$

$$B = \frac{20.2}{1.21} = \frac{40}{21}$$

$$B = \frac{40}{21}$$

Vậy .

Câu 14. (HSG 7 huyện Sơn Trà 2018 - 2019; huyện Bố Trạch; huyện Cẩm Phả 2017 - 2018; huyện Thiệu Hóa 2016 - 2017; huyện Triệu Sơn 2015 - 2016)

$$A = \frac{1}{2} + \frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} + \dots + \frac{1}{2015.2017}$$

Tính giá trị các biểu thức sau:

Lời giải

$$A = \frac{1}{2} + \frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} + \dots + \frac{1}{2015.2017}$$

$$= \frac{1}{2} \cdot \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{3}{2} \cdot \frac{3}{4} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \dots \cdot \frac{2016}{2015} \cdot \frac{2016}{2017}$$

$$= \frac{1}{2} \cdot \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{3}{2} \cdot \frac{3}{4} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \dots \cdot \frac{2016}{2015} \cdot \frac{2016}{2017} = \frac{2016}{2017}$$

Câu 15. (HSG 7 Trường Đông Xuân, TP Phúc Yên, Vĩnh Lộc; huyện Trục Ninh 2022 - 2023)

$$C = \frac{1}{2} + \frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} + \dots + \frac{1}{2021.2023}$$

Tính giá trị biểu thức:

Lời giải

$$C = \frac{1}{2} + \frac{1}{1.3} + \frac{1}{2.4} + \frac{1}{3.5} + \dots + \frac{1}{2021.2023}$$

$$C = \frac{1}{2} \cdot \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{3}{2} \cdot \frac{3}{4} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \dots \cdot \frac{2022}{2021} \cdot \frac{2022}{2023}$$

$$C = \frac{1}{2} \cdot \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{3}{2} \cdot \frac{3}{4} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \dots \cdot \frac{2022}{2021} \cdot \frac{2022}{2023}$$

$$C = \frac{1}{2} \cdot \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{3}{2} \cdot \frac{3}{4} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \dots \cdot \frac{2022}{2021} \cdot \frac{2022}{2023}$$

$$= \frac{1}{2} \cdot \frac{2022}{1} \cdot \frac{2}{2023} = \frac{2022}{2023}$$

$$C = \frac{2022}{2023}$$

Vậy .

Câu 16. (HSG 7 huyện Nga Sơn 2016 - 2017)

$$C = \frac{1}{1} - \frac{1}{3} + \frac{1}{6} - \frac{1}{10} + \frac{1}{15} - \dots - \frac{1}{210}$$

Thực hiện phép tính

Lời giải

$$\begin{aligned} C &= \frac{1}{1} - \frac{1}{3} + \frac{1}{6} - \frac{1}{10} + \frac{1}{15} - \dots - \frac{1}{210} \\ &= \frac{2}{3} \cdot \frac{5}{6} \cdot \frac{9}{10} \cdot \frac{14}{15} \dots \frac{209}{210} \\ &= \frac{4}{6} \cdot \frac{10}{12} \cdot \frac{18}{20} \cdot \frac{28}{30} \dots \frac{418}{420} = \frac{(1.4)(2.5)(3.6)(4.7) \dots (19.22)}{(2.3)(3.4)(4.5)(5.6) \dots (20.21)} \\ &= \frac{(1.2.3 \dots 19)(4.5.6.7 \dots 22)}{(2.3.4 \dots 20)(3.4.5.6 \dots 21)} = \frac{11}{30} \end{aligned}$$

Câu 17. (HSG 7 huyện Trực Ninh 2022 - 2023; huyện Việt Yên 2018 - 2019)

$$A = \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \dots - \frac{1}{100} + \frac{1}{121} - \frac{1}{144}$$

Tính

Lời giải

$$\begin{aligned} A &= \frac{1}{4} - \frac{1}{8} + \frac{1}{16} - \frac{1}{32} + \dots - \frac{1}{100} + \frac{1}{121} - \frac{1}{144} \\ &= \frac{-3}{4} \cdot \frac{-8}{9} \cdot \frac{-15}{16} \dots \frac{-99}{100} \cdot \frac{-120}{121} \\ &= \frac{1.3}{2^2} \cdot \frac{2.4}{3^2} \cdot \frac{3.5}{4^2} \dots \frac{9.11}{10^2} \cdot \frac{10.12}{11^2} \\ &= \frac{1.2.3.4 \dots 10.3.4.5.6 \dots 11.12}{2^2.3^2 \dots 11^2} \\ &= \frac{1.2.11.12}{2^2.11^2} = \frac{12}{22} = \frac{6}{11} \end{aligned}$$

Câu 18. (HSG 7 huyện Thanh Miện 2022 - 2023)

$$C = \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \frac{1}{5^2} + \dots - \frac{1}{100^2} + \frac{1}{101^2} - \frac{1}{102^2}$$

Thực hiện phép tính:

Lời giải

$$\begin{aligned} C &= \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} - \frac{1}{5^2} + \dots - \frac{1}{100^2} + \frac{1}{101^2} - \frac{1}{102^2} \\ &= \frac{-3}{2^2} \cdot \frac{-8}{3^2} \cdot \frac{-15}{4^2} \dots \frac{-9999}{100^2} \cdot \frac{-10200}{101^2} \end{aligned}$$

$$\begin{aligned}
 &= \frac{-1.3}{2^2} \cdot \frac{-2.4}{3^2} \cdot \frac{-3.5}{4^2} \cdot \frac{1}{4} \cdot \frac{-99.101}{100^2} \cdot \frac{-100.102}{101^2} \\
 &= \frac{-1.(-2).(-3)....(-100).3.4.5....102}{2.3.4....101.2.3.4....101} \\
 &= \frac{1.2.3....100.3.4.5....102}{2.3.4....101.2.3.4....101} = \frac{1.102}{101.2} = \frac{51}{101}
 \end{aligned}$$

Câu 19. (HSG 7 huyện Nga Sơn – Thanh Hóa; Tp Thủ Đức 2022 - 2023)

$$C = \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - \frac{1}{2023^2} - \frac{1}{2}$$

Thực hiện phép tính:

Lời giải

$$C = \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - \frac{1}{2023^2} - \frac{1}{2}$$

$$C = \frac{1-2^2}{2^2} \times \frac{1-3^2}{3^2} \times \frac{1-4^2}{4^2} \cdot \frac{1}{4} \cdot \frac{1-2023^2}{2023^2}$$

$$C = \frac{-3}{2^2} \times \frac{-8}{3^2} \times \frac{-15}{4^2} \cdot \frac{1}{4} \cdot \frac{-4092528}{2023^2}$$

$$C = \frac{3}{2^2} \times \frac{8}{3^2} \times \frac{15}{4^2} \cdot \frac{1}{4} \cdot \frac{4092528}{2023^2}$$

$$C = \frac{1.3}{2^2} \times \frac{2.4}{3^2} \times \frac{3.5}{4^2} \cdot \frac{1}{4} \cdot \frac{2022 \times 2024}{2023^2}$$

$$C = \frac{1 \times 2 \times 3 \times 4 \times 5 \cdot \frac{1}{4} \cdot 2022 \times 2024}{2^2 \times 3^2 \times 4^2 \cdot \frac{1}{4} \cdot 2023^2}$$

$$C = \frac{(1.2.3.4.5 \cdot \frac{1}{4} \cdot 2022)(3.4.5 \cdot \frac{1}{4} \cdot 99.2024)}{(2.3.4.5 \cdot \frac{1}{4} \cdot 99.2023)(2.3.4.5 \cdot \frac{1}{4} \cdot 2023)} = \frac{1.2024}{2023.2} = \frac{1012}{2023}$$

$$C = \frac{1012}{2023}$$

Vậy

Câu 20. (HSG 7 tỉnh Thái Bình 2022 - 2023)

$$B = \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - \frac{1}{100^2} - \frac{1}{2}$$

Cho

Hãy so sánh B với $-\frac{1}{2}$

Lời giải

$$B = \frac{1}{2^2} - \frac{1}{3^2} - \frac{1}{4^2} - \frac{1}{100^2} - \frac{1}{2}$$

$$= \frac{-3}{2^2} \cdot \frac{-8}{3^2} \cdot \frac{-15}{4^2} \cdot \frac{1}{4} \cdot \frac{-9999}{100^2}$$

$$= \frac{-1.3}{2^2} \cdot \frac{-2.4}{3^2} \cdot \frac{-3.5}{4^2} \cdot \frac{1}{4} \cdot \frac{-99.101}{100^2}$$

$$= \frac{-1.(-2).(-3)....(-99).3.4.5....101}{2.3.4....101.2.3.4....101}$$

$$= - \frac{1.2.3....99.3.4.5....101}{2.3.4....100.2.3.4....100} = - \frac{1.101}{100.2} = - \frac{101}{200}$$

$$\text{Vì } \frac{101}{200} > \frac{100}{200} = \frac{1}{2} \Rightarrow - \frac{101}{200} < - \frac{1}{2} \quad \text{hay} \quad B < - \frac{1}{2}$$

$$\text{Vậy } B < - \frac{1}{2}.$$

Câu 21. (HSG 7 TP Sầm Sơn, tỉnh Thanh Hoá 2022 - 2023)

$$B = \frac{1}{15} - \frac{1}{150} - \frac{1}{210} - \frac{1}{280} - \frac{1}{210}$$

Thực hiện phép tính:

Lời giải

$$B = \frac{1}{15} - \frac{1}{150} - \frac{1}{210} - \frac{1}{280} - \frac{1}{210}$$

$$= \frac{14}{15} \times \frac{20}{21} \times \frac{27}{28} \times \frac{209}{210} = \frac{28}{30} \times \frac{40}{42} \times \frac{54}{56} \times \frac{418}{420} = \frac{4 \times 5 \times 9 \times 22}{5 \times 7 \times 7 \times 20 \times 21} = \frac{11}{15}$$

Câu 22. (HSG 7 huyện Hưng Hà, tỉnh, trường Minh Khai 2022 - 2023)

$$B = \frac{(1+17)\frac{1}{18} + \frac{17}{2} \frac{1}{18} + \frac{17}{3} \frac{1}{18} + \frac{17}{19} \frac{1}{18}}{(1+19)\frac{1}{20} + \frac{19}{2} \frac{1}{20} + \frac{19}{3} \frac{1}{20} + \frac{19}{17} \frac{1}{20}}$$

Tính giá trị của các biểu thức:

Lời giải

$$B = \frac{(1+17)\frac{1}{18} + \frac{17}{2} \frac{1}{18} + \frac{17}{3} \frac{1}{18} + \frac{17}{19} \frac{1}{18}}{(1+19)\frac{1}{20} + \frac{19}{2} \frac{1}{20} + \frac{19}{3} \frac{1}{20} + \frac{19}{17} \frac{1}{20}} = \frac{18 \cdot \frac{19}{2} \cdot \frac{20}{3} \cdot \frac{21}{4} \cdot \frac{36}{19}}{20 \cdot \frac{21}{2} \cdot \frac{22}{3} \cdot \frac{23}{4} \cdot \frac{36}{17}}$$

$$= \frac{18.19.20.21....36}{1.2.3....17.18.19} \cdot \frac{1.2.3....17}{20.21.22....36}$$

$$= \frac{20.21.....36}{1.2.3....17} \cdot \frac{1.2.3....17}{20.21.22....36}$$

$$= 1$$

Câu 23. (HSG 7 huyện Cẩm Thủy, tỉnh Thanh Hóa, 2021 - 2022)

$$P = \frac{1}{9} + \frac{7}{90} + \frac{7}{200} + \frac{7}{330} + \frac{7}{2900}$$

Thực hiện phép tính:

Lời giải

$$P = \frac{1}{9} + \frac{7}{90} + \frac{7}{200} + \frac{7}{330} + \frac{7}{2900}$$

$$= \frac{16}{9} \cdot \frac{27}{20} \cdot \frac{40}{43} \cdot \frac{2907}{2900} = \frac{2.8.3.9.4.10.....51.57}{1.9.2.10.3.11.....50.58}$$

$$= \frac{51.8}{58} = \frac{204}{29}$$

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