Mathscope

translated by Pham Van Thuan, Eckard Specht

Vol I, Problems in Mathematics Journal for the Youth

Mathscope is a free problem resource selected from mathematical problem solving journals in Vietnam. This freely accessible collection is our effort to introduce elementary mathematics problems to foreign friends for either recreational or professional use. We would like to give you a new taste of Vietnamese mathematical culture.

It's now not too hard to find problems and solutions on the Internet due to the increasing number of websites devoted to mathematical problem solving. It is our hope that this collection saves you considerable time searching the problems you really want. We intend to give an outline of solutions to the problems in the future. Now enjoy these "cakes" from Vietnam first.

153.1 (Nguyễn Đông Yên) Prove that if $y \geq y^3 + x^2 + |x| + 1$, then $x^2 + y^2 \geq 1$. Find all pairs of (x, y) such that the first inequality holds while equality in the second one attains.

153.2 (Tạ Văn Tự) Given natural numbers m, n, and a real number $a > 1$, prove the inequality

$$a^{\frac{2n}{m}} - 1 \geq n(a^{\frac{n+1}{m}} - a^{\frac{n-1}{m}}).$$

153.3 (Nguyễn Minh Đức) Prove that for each $0 < \epsilon < 1$, there exists a natural number n_0 such that the coefficients of the polynomial

$$(x+y)^n(x^2 - (2-\epsilon)xy + y^2)$$

are all positive for each natural number $n \geq n_0$.

200.1 (Phạm Ngọc Quang) In a triangle ABC, let $BC = a$, $CA = b$, $AB = c$, I be the incenter of the triangle. Prove that

$$a.IA^2 + b.IB^2 + c.IC^2 = abc.$$

200.2 (Trần Xuân Đáng) Let $a, b, c \in \mathbb{R}$ such that $a + b + c = 1$, prove that

$$15(a^3 + b^3 + c^3 + ab + bc + ca) + 9abc \geq 7.$$

200.3 (Đặng Hùng Thắng) Let a, b, c be integers such that the quadratic function $ax^2 + bx + c$ has two distinct zeros in the interval $(0, 1)$. Find the least value of a, b, and c.

200.4 (Nguyễn Đăng Phất) A circle is tangent to the circumcircle of a triangle ABC and also tangent to side AB, AC at P, Q respectively. Prove that the midpoint of PQ is the incenter of triangle ABC. With edge and compass, construct the circle tangent to sides AB and AC and to the circle (ABC).

200.5 (Nguyễn Văn Mậu) Let $x, y, z, t \in [1, 2]$, find the smallest positive possible p such that the inequality holds

$$\frac{y+t}{x+z} + \frac{z+t}{t+x} \leq p\left(\frac{y+z}{x+y} + \frac{x+z}{y+t}\right).$$

200.6 (Nguyễn Minh Hà) Let a, b, c be real positive numbers such that $a + b + c = \pi$, prove that $\sin a + \sin b + \sin c + \sin(a+b+c) \leq \sin(a+b) + \sin(b+c) + \sin(c+a)$.

208.1 (Đặng Hùng Thắng) Let $a_1, a_2, \ldots, a_n$ be the odd numbers, none of which has a prime divisors greater than 5, prove that

$$\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n} < \frac{15}{8}.$$

208.2 (Trần Văn Vuông) Prove that if r, and s are real numbers such that $r^3 + s^3 > 0$, then the equation $x^3 + 3rx - 2s = 0$ has a unique solution

$$x = \sqrt[3]{s + \sqrt{s^2 + r^3}} + \sqrt[3]{s - \sqrt{s^2 - r^3}}.$$

Using this result to solve the equations $x^3 + x + 1 = 0$, and $20x^3 - 15x^2 - 1 = 0$.

209.1 (Đặng Hùng Thắng) Find integer solutions (x, y) of the equation

$$(x^2 + y)(x + y^2) = (x - y)^3.$$

209.2 (Trần Duy Hinh) Find all natural numbers n such that $n^{n+1} + (n+1)^n$ is divisible by 5.

209.3 (Đào Trường Giang) Given a right triangle with hypotenuse BC, the incircle of the triangle is tangent to the sides AB amd BC respectively at P, and Q. A line through the incenter and the midpoint F of AC intersects side AB at E; the line through P and Q meets the altitude AH at M. Prove that $AM = AE$.

213.1 (Hồ Quang Vinh) Let a, b, c be positive real numbers such that $a + b + c = 2r$, prove that

$$\frac{ab}{r-c} + \frac{bc}{r-a} + \frac{ca}{r-b} \geq 4r.$$

213.2 (Phạm Văn Hùng) Let ABC be a triangle with altitude AH, let M, N be the midpoints of AB and AC. Prove that the circumcircles of triangles HBM, HCN, amd AMN has a common point K, prove that the extended HK is through the midpoint of MN.

213.3 (Nguyễn Minh Đức) Given three sequences of numbers $\{x_n\}_{n=0}^{\infty}$, $\{y_n\}_{n=0}^{\infty}$, $\{z_n\}_{n=0}^{\infty}$ such that x_0, y_0, z_0 are positive, $x_{n+1} = y_n + \frac{1}{z_n}$, $y_{n+1} = z_n + \frac{1}{x_n}$, $z_{n+1} = x_n + \frac{1}{y_n}$ for all $n \geq 0$. Prove that there exist positive numbers s and t such that $s\sqrt{n} \leq x_n \leq t\sqrt{n}$ for all $n \geq 1$.

216.1 (Thới Ngọc Ánh) Solve the equation

$$(x+2)^2 + (x+3)^3 + (x+4)^4 = 2.$$

216.2 (Lê Quốc Hán) Denote by (O, R), (I, R_a) the circumcircle, and the excircle of angle A of triangle ABC. Prove that

$$IA.IB.IC = 4R.R_a^2.$$

216.3 (Nguyễn Đễ) Prove that if $-1 < a < 1$ then

$$\sqrt[4]{1-a^2} + \sqrt[4]{1-a} + \sqrt[4]{1+a} < 3.$$

216.4 (Trần Xuân Đáng) Let (x_n) be a sequence such that $x_1 = 1$, $(n+1)(x_{n+1} - x_n) \geq 1 + x_n$, $\forall n \geq 1$, $n \in \mathbb{N}$. Prove that the sequence is not bounded.

216.5 (Hoàng Đức Tân) Let P be any point interior to triangle ABC, let d_A, d_B, d_C be the distances of P to the vertice A, B, C respectively. Denote by p, q, r distances of P to the sides of the triangle. Prove that

$$d_A^2 \sin^2 A + d_B^2 \sin^2 B + d_C^2 \sin^2 C \leq 3(p^2 + q^2 + r^2).$$

220.1 (Trần Duy Hinh) Does there exist a triple of distinct numbers a, b, c such that

$$(a-b)^5 + (b-c)^5 + (c-a)^5 = 0.$$

220.2 (Phạm Ngọc Quang) Find triples of three non-negative integers (x, y, z) such that $3x^2 + 54 = 2y^2 + 4z^2$, $5x^2 + 74 = 3y^2 + 7z^2$, and $x + y + z$ is a minimum.

220.3 (Đặng Hùng Thắng) Given a prime number p and positive integer a, $a \leq p$, suppose that $A = \sum_{k=0}^{p-1} a^k$. Prove that for each prime divisor q of A, we have $q - 1$ is divisible by p.

220.4 (Ngọc Đạm) The bisectors of a triangle ABC meet the opposite sides at D, E, F. Prove that the necessary and sufficient condition in order for triangle ABC to be equilateral is

$$\text{Area}(DEF) = \frac{1}{4}\text{Area}(ABC).$$

220.5 (Phạm Hiến Bằng) In a triangle ABC, denote by l_a, l_b, l_c the internal angle bisectors, m_a, m_b, m_c the medians, and h_a, h_b, h_c the altitudes to the sides a, b, c of the triangle. Prove that

$$\frac{m_a}{l_b + h_b} + \frac{m_b}{l_c + h_c} + \frac{m_c}{l_a + h_a} \geq \frac{3}{2}.$$

220.6 (Nguyễn Hữu Thảo) Solve the system of equations

$$\begin{aligned} x^2 + y^2 + xy &= 37, \\ x^2 + z^2 + zx &= 28, \\ y^2 + z^2 + yz &= 19. \end{aligned}$$

221.1 (Ngô Hân) Find the greatest possible natural number n such that 1995 is equal to the sum of n numbers $a_1, a_2, \ldots, a_n$, where a_i, $(i = 1, 2, \ldots, n)$ are composite numbers.

221.2 (Trần Duy Hinh) Find integer solutions (x, y) of the equation $x(1+x+x^2) = 4y(y+1)$.

221.3 (Hoàng Ngọc Cảnh) Given a triangle with incenter I, let ℓ be variable line passing through I. Let ℓ intersect the ray CB, sides AC, AB at M, N, P respectively. Prove that the value of

$$\frac{AB}{PA.PB} + \frac{AC}{NA.NC} - \frac{BC}{MB.MC}$$

is independent of the choice of ℓ.

221.4 (Nguyễn Đức Tấn) Given three integers x, y, z such that $x^4 + y^4 + z^4 = 1984$, prove that $p = 20^x + 11^y - 1996^z$ can not be expressed as the product of two consecutive natural numbers.

221.5 (Nguyễn Lê Dũng) Prove that if $a, b, c > 0$ then

$$\frac{a^2+b^2}{a+b} + \frac{b^2+c^2}{b+c} + \frac{c^2+a^2}{c+a} \le \frac{3(a^2+b^2+c^2)}{a+b+c}.$$

221.6 (Trịnh Bằng Giang) Let I be an interior point of triangle ABC. Lines IA, IB, IC meet BC, CA, AB respectively at A', B', C'. Find the locus of I such that

$$(IAC')^2 + (IBA')^2 + (ICB')^2 = (IBC')^2 + (ICA')^2 + (IAB')^2,$$

where $(.)$ denotes the area of the triangle.

221.7 (Hồ Quang Vinh) The sequences $(a_n)_{n\in\mathbb{N}^*}$, $(b_n)_{n\in\mathbb{N}^*}$ are defined as follows

$$a_n = 1 + \frac{n(1+n)}{1+n^2} + \cdots + \frac{n^n(1+n^n)}{1+n^{2n}}$$
$$b_n = \left(\frac{a_n}{n+1}\right)^{\frac{1}{n(n+1)}}, \ \forall n \in \mathbb{N}^*.$$

Find $\lim\limits_{n\to\infty} b_n$.

230.1 (Trần Nam Dũng) Let $m \in \mathbb{N}, m \ge 2, p \in \mathbb{R}, 0 < p < 1$. Let $a_1, a_2, \ldots, a_m$ be real positive numbers. Put $s = \sum\limits_{i=1}^{m} a_i$. Prove that

$$\sum_{i=1}^{m} \left(\frac{a_i}{s-a_i}\right)^p \ge \frac{1}{1-p}\left(\frac{1-p}{p}\right)^p,$$

with equality if and only if $a_1 = a_2 = \cdots = a_m$ and $m(1-p) = 1$.

235.1 (Đặng Hùng Thắng) Given real numbers x, y, z such that

$$\begin{aligned} a+b &= 6, \\ ax+by &= 10, \\ ax^2+by^2 &= 24, \\ ax^3+by^3 &= 62, \end{aligned}$$

determine ax^4+by^4.

235.2 (Hà Đức Vượng) Let ABC be a triangle, let D be a fixed point on the opposite ray of ray BC. A variable ray D_x intersects the sides AB, AC at E, F, respectively. Let M and N be the midpoints of BF, CE, respectively. Prove that the line MN has a fixed point.

235.3 (Đàm Văn Nhỉ) Find the maximum value of

$$\frac{a}{bcd+1}+\frac{b}{cda+1}+\frac{c}{dab+1}+\frac{d}{abc+1},$$

where $a, b, c, d \in [0,1]$.

235.4 (Trần Nam Dũng) Let M be any point in the plane of an equilateral triangle ABC. Denote by x, y, z the distances from P to the vertices and p, q, r the distances from M to the sides of the triangle. Prove that

$$p^2+q^2+r^2 \geq \frac{1}{4}(x^2+y^2+z^2),$$

and that this inequality characterizes all equilateral triangles in the sense that we can always choose a point M in the plane of a non-equilateral triangle such that the inequality is not true.

241.1 (Nguyễn Khánh Trình, Trần Xuân Đáng) Prove that in any acute triangle ABC, we have the inequality

$$\sin A \sin B+\sin B \sin C+\sin C \sin A \leq (\cos A+\cos B+\cos C)^2.$$

241.2 (Trần Nam Dũng) Given n real numbers $x_1, x_2, \ldots, x_n$ in the interval $[0,1]$, prove that

$$\left[\frac{n}{2}\right] \geq x_1(1-x_2)+x_2(1-x_3)+\cdots+x_{n-1}(1-x_n)+x_n(1-x_1).$$

241.3 (Trần Xuân Đáng) Prove that in any acute triangle ABC

$$\sin A \sin B+\sin B \sin C+\sin C \sin A \geq (1+\sqrt{2\cos A \cos B \cos C})^2.$$

242.1 (Phạm Hữu Hoài) Let α, β, γ real numbers such that $\alpha \le \beta \le \gamma$, $\alpha < \beta$. Let $a, b, c \in [\alpha, \beta]$ sucht that $a + b + c = \alpha + \beta + \gamma$. Prove that

$$a^2 + b^2 + c^2 \le \alpha^2 + \beta^2 + \gamma^2.$$

242.2 (Lê Văn Bảo) Let p and q be the perimeter and area of a rectangle, prove that

$$p \ge \frac{32q}{2q + p + 2}.$$

242.3 (Tô Xuân Hải) In triangle ABC with one angle exceeding $\frac{2}{3}\pi$, prove that

$$\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} \ge 4 - \sqrt{3}.$$

243.1 (Ngô Đức Minh) Solve the equation

$$\sqrt{4x^2 + 5x + 1} - 2\sqrt{x^2 - x + 1} = 9x - 3.$$

243.2 (Trần Nam Dũng) Given $2n$ real numbers $a_1, a_2, \ldots, a_n; b_1, b_2, \ldots, b_n$, suppose that $\sum\limits_{j=1}^{n} a_j \ne 0$ and $\sum\limits_{j=1}^{n} b_j \ne 0$. Prove that the following inequality

$$\sum_{j=1}^{n} a_j b_j + \left\{ \left(\sum_{j=1}^{n} a_j^2 \right) \left(\sum_{j=1}^{n} b_j^2 \right) \right\}^{\frac{1}{2}} \ge \frac{2}{n} \left(\sum_{j=1}^{n} a_j \right) \left(\sum_{j=1}^{n} b_j \right),$$

with equaltiy if and only if

$$\frac{a_i}{\sum\limits_{j=1}^{n} a_j} + \frac{b_i}{\sum_{j=1}^{n} b_j} = \frac{2}{n}, \quad i = 1, 2, \ldots, n.$$

243.3 (Hà Đức Vượng) Given a triangle ABC, let AD and AM be the internal angle bisector and median of the triangle respectively. The circumcircle of ADM meet AB and AC at E, and F respectively. Let I be the midpoint of EF, and N, P be the intersections of the line MI and the lines AB and AC respectively. Determine, with proof, the shape of the triangle ANP.

243.4 (Tô Xuân Hải) Prove that

$$\arctan\frac{1}{5} + \arctan 2 + \arctan 3 - \arctan\frac{1}{239} = \pi.$$

243.5 (Huỳnh Minh Việt) Given real numbers x, y, z such that $x^2 + y^2 + z^2 = k$, $k > 0$, prove the inequality

$$\frac{2}{k}xyz - \sqrt{2k} \le x + y + z \le \frac{2}{k}xyz + \sqrt{2k}.$$

244.1 (Thái Viết Bảo) Given a triangle ABC, let D and E be points on the sides AB and AC, respectively. Points M, N are chosen on the line segment DE such that $DM = MN = NE$. Let BC intersect the rays AM and AN at P and Q, respectively. Prove that if $BP < PQ$, then $PQ < QC$.

244.2 (Ngô Văn Thái) Prove that if $0 < a, b, c \leq 1$, then

$$\frac{1}{a+b+c} \geq \frac{1}{3} + (1-a)(1-b)(1-c).$$

244.3 (Trần Chí Hòa) Given three positive real numbers x, y, z such that $xy + yz + zx + \frac{2}{a}xyz = a^2$, where a is a given positive number, find the maximum value of $c(a)$ such that the inequality $x + y + z \geq c(a)(xy + yz + zx)$ holds.

244.4 (Đàm Văn Nhỉ) The sequence $\{p(n)\}$ is recursively defined by

$$p(1) = 1, \quad p(n) = 1p(n-1) + 2p(n-2) + \cdots + (n-1)p(n-1)$$

for $n \geq 2$. Determine an explicit formula for $n \in \mathbb{N}^*$.

244.5 (Nguyễn Vũ Lương) Solve the system of equations

$$4xy + 4(x^2 + y^2) + \frac{3}{(x+y)^2} = \frac{85}{3},$$
$$2x + \frac{1}{x+y} = \frac{13}{3}.$$

248.1 (Trần Văn Vương) Given three real numbers x, y, z such that $x \geq 4$, $y \geq 5$, $z \geq 6$ and $x^2 + y^2 + z^2 \geq 90$, prove that $x + y + z \geq 16$.

248.2 (Đỗ Thanh Hân) Solve the system of equations

$$x^3 - 6z^2 + 12z - 8 = 0,$$
$$y^3 - 6x^2 + 12x - 8 = 0,$$
$$z^3 - 6y^2 + 12y - 8 = 0.$$

248.3 (Phương Tố Tử) Let the incircle of an equilateral triangle ABC touch the sides AB, AC, BC respectively at C', B' and A'. Let M be any point on the minor arc $B'C'$, and H, K, L the orthogonal projections of M onto the sides BC, AC and AB, respectively. Prove that

$$\sqrt{MH} = \sqrt{MK} + \sqrt{ML}.$$

250.1 (Đặng Hùng Thắng) Find all pairs (x, y) of natural numbers $x > 1$, $y > 1$, such that $3x + 1$ is divisible by y and simultaneously $3y + 1$ is divisible by x.

250.2 (Nguyễn Ngọc Khoa) Prove that there exists a polynomial with integer coefficients such that its value at each root t of the equation $t^8 - 4t^4 + 1 = 0$ is equal to the value of

$$f(t) = \frac{5t^2}{t^8 + t^5 - t^3 - 5t^2 - 4t + 1}$$

for this value of t.

250.3 (Nguyễn Khắc Minh) Consider the equation $f(x) = ax^2 + bx + c$ where $a < b$ and $f(x) \geq 0$ for all real x. Find the smallest possible value of

$$p = \frac{a+b+c}{b-a}.$$

250.4 (Trần Đức Thịnh) Given two fixed points B and C, let A be a variable point on the semiplanes with boundary BC such that A, B, C are not collinear. Points D, E are chosen in the plane such that triangles ADB and AEC are right isosceles and $AD = DB$, $EA = EC$, and D, C are on different sides of AB; B, E are on different sides of AC. Let M be the midpoint of DE, prove that line AM has a fixed point.

250.5 (Trần Nam Dũng) Prove that if $a, b, c > 0$ then

$$\frac{1}{2} + \frac{a^2+b^2+c^2}{ab+bc+ca} \geq \frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{1}{2}\Big(4 - \frac{ab+bc+ca}{a^2+b^2+c^2}\Big).$$

250.6 (Phạm Ngọc Quang) Given a positive integer m, show that there exist prime integers a, b such that the following conditions are simultaneously satisfied:

$$|a| \leq m, \quad |b| \leq m \quad \text{and} \quad 0 < a + b\sqrt{2} \leq \frac{1+\sqrt{2}}{m+2}.$$

250.7 (Lê Quốc Hán) Given a triangle ABC such that $\cot A, \cot B$ and $\cot C$ are respectively terms of an arithmetic progression. Prove that $\angle GAC = \angle GBA$, where G is the centroid of the triangle.

250.8 (Nguyễn Minh Đức) Find all polynomials with real coefficients $f(x)$ such that $\cos(f(x))$, $x \in \mathbb{R}$, is a periodic function.

251.1 (Nguyễn Duy Liên) Find the smallest possible natural number n such that $n^2 + n + 1$ can be written as a product of four prime numbers.

251.2 (Nguyễn Thanh Hải) Given a cubic equation

$$x^3 - px^2 + qx - p = 0,$$

where $p, q \in \mathbb{R}^*$, prove that if the equation has only real roots, then the inequality

$$p \geq \Big(\frac{1}{4} + \frac{\sqrt{2}}{8}\Big)(q+3)$$

holds.

251.3 (Nguyễn Ngọc Bình Phương) Given a circle with center O and radius r inscribed in triangle ABC. The line joining O and the midpoint of side BC intersects the altitude from vertex A at I. Prove that $AI = r$.

258.1 (Đặng Hùng Thắng) Let a, b, c be positive integers such that

$$a^2 + b^2 = c^2(1 + ab),$$

prove that $a \geq c$ and $b \geq c$.

258.2 (Nguyễn Việt Hải) Let D be any point between points A and B. A circle Γ is tangent to the line segment AB at D. From A and B, two tangents to the circle are drawn, let E and F be the points of tangency, respectively, D distinct from E, F. Point M is the reflection of A across E, point N is the reflection of B across F. Let EF intersect AN at K, BM at H. Prove that triangle DKH is isosceles, and determine the center of Γ such that $\triangle DKH$ is equilateral.

258.3 (Vi Quốc Dũng) Let AC be a fixed line segment with midpoint K, two variable points B, D are chosen on the line segment AC such that K is the midpoint of BD. The bisector of angle $\angle BCD$ meets lines AB and AD at I and J, respectively. Suppose that M is the second intersection of circumcircle of triangle ABD and AIJ. Prove that M lies on a fixed circle.

258.4 (Đặng Kỳ Phong) Find all functions $f(x)$ that satisfy simultaneously the following conditions

i) $f(x)$ is defined and continuous on $\mathbb{R}$;

ii) for each set of 1997 numbers $x_1, x_2, ..., x_{1997}$ such that $x_1 < x_2 < \cdots < x_n$, the inequality

$$f(x_{999}) \geq \frac{1}{1996}\big(f(x_1) + f(x_2) + \cdots + f(x_{998}) + f(x_{1000}) + f(x_{1001}) + \cdots + f(x_{1997})\big).$$

holds.

259.1 (Nguyễn Phước) Solve the equation

$$(x + 3\sqrt{x} + 2)(x + 9\sqrt{x} + 18) = 168x.$$

259.2 (Viên Ngọc Quang) Given four positive real numbers a, b, c and d such that the quartic equation $ax^4 - ax^3 + bx^2 - cx + d = 0$ has four roots in the interval $(0, \frac{1}{2})$, the roots not being necessarily distinct. Prove that

$$21a + 164c \geq 80b + 320d.$$

259.3 (Hồ Quang Vinh) Given is a triangle ABC. The excircle of ABC inside angle A touches side BC at A_1, and the other two excircles inside angles B, C touch sides CA and AB at B_1, C_1, respectively. The lines AA_1, BB_1, CC_1 are concurrent at point N. Let D, E, F be the orthogonal projections of N onto the sides BC, CA and AB, respectively. Suppose that R is the circumradius and r the inradius of triangle ABC. Denote by $S(XYZ)$ the area of triangle XYZ, prove that

$$\frac{S(DEF)}{S(ABC)} = \frac{r}{R}\left(1 - \frac{r}{R}\right).$$

261.1 (Hồ Quang Vinh) Given a triangle ABC, its internal angle bisectors BE and CF, and let M be any point on the line segment EF. Denote by S_A, S_B, and S_C the areas of triangles MBC, MCA, and MAB, respectively. Prove that

$$\frac{\sqrt{S_B} + \sqrt{S_C}}{\sqrt{S_A}} \le \sqrt{\frac{AC + AB}{BC}},$$

and determine when equality holds.

261.2 (Editorial Board) Find the maximum value of the expression

$$A = 13\sqrt{x^2 - x^4} + 9\sqrt{x^2 + x^4} \qquad \text{for} \quad 0 \le x \le 1.$$

261.3 (Editorial Board) The sequence (a_n), $n = 1, 2, 3, \ldots$, is defined by $a_1 > 0$, and $a_{n+1} = ca_n^2 + a_n$ for $n = 1, 2, 3, \ldots$, where c is a constant. Prove that

a) $a_n \ge \sqrt{c^{n-1} n^n a_1^{n+1}}$, and

b) $a_1 + a_2 + \cdots + a_n > n\left(na_1 - \frac{1}{c}\right)$ for $n \in \mathbb{N}$.

261.4 (Editorial Board) Let X, Y, Z be the reflections of A, B, and C across the lines BC, CA, and AB, respectively. Prove that X, Y, and Z are collinear if and only if

$$\cos A \cos B \cos C = -\frac{3}{8}.$$

261.5 (Vinh Competition) Prove that if $x, y, z > 0$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ then the following inequality holds:

$$\left(1 - \frac{1}{1+x^2}\right)\left(1 - \frac{1}{1+y^2}\right)\left(1 - \frac{1}{1+z^2}\right) > \frac{1}{2}.$$

261.6 (Đỗ Văn Đức) Given four real numbers x_1, x_2, x_3, x_4 such that $x_1 + x_2 + x_3 + x_4 = 0$ and $|x_1| + |x_2| + |x_3| + |x_4| = 1$, find the maximum value of $\prod_{1 \le i < j \le 4} (x_i - x_j)$.

261.7 (Đoàn Quang Mạnh) Given a rational number $x \geq 1$ such that there exists a sequence of integers (a_n), $n = 0, 1, 2, \ldots$, and a constant $c \neq 0$ such that $\lim_{n\to\infty}(cx^n - a_n) = 0$. Prove that x is an integer.

262.1 (Ngô Văn Hiệp) Let ABC an equilateral triangle of side length a. For each point M in the interior of the triangle, choose points D, E, F on the sides CA, AB, and BC, respectively, such that $DE = MA$, $EF = MB$, and $FD = MC$. Determine M such that $\triangle DEF$ has smallest possible area and calculate this area in terms of a.

262.2 (Nguyễn Xuân Hùng) Given is an acute triangle with altitude AH. Let D be any point on the line segment AH not coinciding with the endpoints of this segment and the orthocenter of triangle ABC. Let ray BD intersect AC at M, ray CD meet AB at N. The line perpendicular to BM at M meets the line perpendicular to CN at N in the point S. Prove that $\triangle ABC$ is isosceles with base BC if and only if S is on line AH.

262.3 (Nguyễn Duy Liên) The sequence (a_n) is defined by

$$a_0 = 2, \quad a_{n+1} = 4a_n + \sqrt{15a_n^2 - 60} \quad \text{for } n \in \mathbb{N}.$$

Find the general term a_n. Prove that $\frac{1}{5}(a_{2n} + 8)$ can be expressed as the sum of squares of three consecutive integers for $n \geq 1$.

262.4 (Tuấn Anh) Let p be a prime, n and k positive integers with $k > 1$. Suppose that $b_i, i = 1, 2, \ldots, k$, are integers such that

i) $0 \leq b_i \leq k-1$ for all i,

ii) p^{nk-1} is a divisor of $\left(\sum_{i=1}^{k} p^{nb_i}\right) - p^{n(k-1)} - p^{n(k-2)} - \cdots - p^n - 1$.

Prove that the sequence $(b_1, b_2, \ldots, b_k)$ is a permutation of the sequence $(0, 1, \ldots, k-1)$.

262.5 (Đoàn Thế Phiệt) Without use of any calculator, determine

$$\sin\frac{\pi}{14} + 6\sin^2\frac{\pi}{14} - 8\sin^4\frac{\pi}{14}.$$

264.1 (Trần Duy Hinh) Prove that the sum of all squares of the divisors of a natural number n is less than $n^2\sqrt{n}$.

264.2 (Hoàng Ngọc Cảnh) Given two polynomials

$$f(x) = x^4 - (1 + e^x) + e^2, \qquad g(x) = x^4 - 1,$$

prove that for distinct positive numbers a, b satisfying $a^b = b^a$, we have $f(a)f(b) < 0$ and $g(a)g(b) > 0$.

264.3 (Nguyễn Phú Yên) Solve the equation

$$\frac{(x-1)^4}{(x^2-3)^2} + (x^2-3)^4 + \frac{1}{(x-1)^2} = 3x^2 - 2x - 5.$$

264.4 (Nguyễn Minh Phương, Nguyễn Xuân Hùng) Let I be the incenter of triangle ABC. Rays AI, BI, and CI meet the circumcircle of triangle ABC again at X, Y, and Z, respectively. Prove that

a) $IX + IY + IZ \geq IA + IB + IC$, b) $\frac{1}{IX} + \frac{1}{IY} + \frac{1}{IZ} \geq \frac{3}{R}$.

265.1 (Vũ Đình Hòa) The lengths of the four sides of a convex quadrilateral are natural numbers such that the sum of any three of them is divisible by the fourth number. Prove that the quadrilateral has two equal sides.

265.2 (Đàm Văn Nhỉ) Let AD, BE, and CF be the internal angle bisectors of triangle ABC. Prove that $p(DEF) \leq \frac{1}{2}p(ABC)$, where $p(XYZ)$ denotes the perimeter of triangle XYZ. When does equality hold?

266.1 (Lê Quang Nẫm) Given real numbers $x, y, z \geq -1$ satisfying $x^3 + y^3 + z^3 \geq x^2 + y^2 + z^2$, prove that $x^5 + y^5 + z^5 \geq x^2 + y^2 + z^2$.

266.2 (Đặng Nhơn) Let $ABCD$ be a rhombus with $\angle A = 120°$. A ray Ax and AB make an angle of $15°$, and Ax meets BC and CD at M and N, respectively. Prove that

$$\frac{3}{AM^2} + \frac{3}{AN^2} = \frac{4}{AB^2}.$$

266.3 (Hà Duy Hưng) Given an isosceles triangle with $\angle A = 90°$. Let M be a variable point on line BC, (M distinct from B, C). Let H and K be the orthogonal projections of M onto lines AB and AC, respectively. Suppose that I is the intersection of lines CH and BK. Prove that the line MI has a fixed point.

266.4 (Lưu Xuân Tình) Let x, y be real numbers in the interval $(0, 1)$ and $x + y = 1$, find the minimum of the expression $x^x + y^y$.

267.1 (Đỗ Thanh Hân) Let x, y, z be real numbers such that

$$x^2 + z^2 = 1,$$
$$y^2 + 2y(x+z) = 6.$$

Prove that $y(z-x) \leq 4$, and determine when equality holds.

267.2 (Vũ Ngọc Minh, Phạm Gia Vĩnh Anh) Let a, b be real positive numbers, x, y, z be real numbers such that

$$x^2 + z^2 = b,$$
$$y^2 + (a-b)y(z+x) = 2ab^2.$$

Prove that $y(z-x) \leq (a+b)b$ with equality if and only if

$$x = \pm\frac{a\sqrt{b}}{\sqrt{a^2+b^2}}, \quad z = \mp\frac{b\sqrt{b}}{\sqrt{a^2+b^2}}, \quad z = \mp\sqrt{b(a^2+b^2)}.$$

267.3 (Lê Quốc Hán) In triangle ABC, medians AM and CN meet at G. Prove that the quadrilateral $BMGN$ has an incircle if and only if triangle ABC is isosceles at B.

267.4 (Trần Nam Dũng) In triangle ABC, denote by a, b, c the side lengths, and F the area. Prove that

$$F \leq \frac{1}{16}(3a^2 + 2b^2 + 2c^2),$$

and determine when equality holds. Can we find another set of the coefficients of a^2, b^2, and c^2 for which equality holds?

268.1 (Đỗ Kim Sơn) In a triangle, denote by a, b, c the side lengths, and let r, R be the inradius and circumradius, respectively. Prove that

$$a(b+c-a)^2 + b(c+a-b)^2 + c(a+b-c)^2 \leq 6\sqrt{3}R^2(2R-r).$$

268.2 (Đặng Hùng Thắng) The sequence (a_n), $n \in \mathbb{N}$, is defined by

$$a_0 = a, \quad a_1 = b, \quad a_{n+2} = da_{n+1} - a_n \qquad \text{for} \quad n = 0, 1, 2, \ldots,$$

where a, b are non-zero integers, d is a real number. Find all d such that a_n is an integer for $n = 0, 1, 2, \ldots$.

271.1 (Đoàn Thế Phiệt) Find necessary and sufficient conditions with respect to m such that the system of equations

$$x^2 + y^2 + z^2 + xy - yz - zx = 1,$$
$$y^2 + z^2 + yz = 2,$$
$$z^2 + x^2 + zx = m$$

has a solution.

272.1 (Nguyễn Xuân Hùng) Given are three externally tangent circles $(O_1), (O_2)$, and (O_3). Let A, B, C be respectively the points of tangency of (O_1) and (O_3), (O_2) and (O_3), (O_1) and (O_2). The common tangent of (O_1) and (O_2) meets C and (O_3) at M and N. Let D be the midpoint of MN. Prove that C is the center of one of the excircles of triangle ABD.

272.2 (Trịnh Bằng Giang) Let $ABCD$ be a convex quadrilateral such that $AB + CD = BC + DA$. Find the locus of points M interior to quadrilateral $ABCD$ such that the sum of the distances from M to AB and CD is equal to the sum of the distances from M to BC and DA.

272.3 (Hồ Quang Vinh) Let M and m be the greatest and smallest numbers in the set of positive numbers $a_1, a_2, \ldots, a_n$, $n \geq 2$. Prove that

$$\Big(\sum_{i=1}^{n} a_i\Big)\Big(\sum_{i=1}^{n} \frac{1}{a_i}\Big) \leq n^2 + \frac{n(n-1)}{2}\Big(\sqrt{\frac{M}{m}} - \sqrt{\frac{m}{M}}\Big)^2.$$

272.4 (Nguyễn Hữu Dự) Find all primes p such that

$$f(p) = (2+3) - (2^2+3^2) + (2^3+3^3) - \cdots - (2^{p-1}+3^{p-1}) + (2^p+3^p)$$

is divisible by 5.

274.1 (Đào Mạnh Thắng) Let p be the semiperimeter and R the circumradius of triangle ABC. Furthermore, let D, E, F be the excenters. Prove that

$$DE^2 + EF^2 + FD^2 \geq 8\sqrt{3}pR,$$

and determine the equality case.

274.2 (Đoàn Thế Phiệt) Detemine the positive root of the equation

$$x \ln\Big(1+\frac{1}{x}\Big)^{1+\frac{1}{x}} - x^3 \ln\Big(1+\frac{1}{x^2}\Big)^{1+\frac{1}{x^2}} = 1 - x.$$

274.3 (N.Khánh Nguyên) Let $ABCD$ be a cyclic quadrilateral. Points M, N, P, and Q are chosen on the sides AB, BC, CD, and DA, respectively, such that $MA/MB = PD/PC = AD/BC$ and $QA/QD = NB/NC = AB/CD$. Prove that MP is perpendicular to NQ.

274.4 (Nguyễn Hào Liễu) Prove the inequality for $x \in \mathbb{R}$:

$$\frac{1 + 2x \arctan x}{2 + \ln(1+x^2)^2} \geq \frac{1 + e^{\frac{x}{2}}}{3 + e^x}.$$

275.1 (Trần Hồng Sơn) Let x, y, z be real numbers in the interval $[-2, 2]$, prove the inequality

$$2(x^6 + y^6 + z^6) - (x^4y^2 + y^4z^2 + z^4x^2) \leq 192.$$

276.1 (Vũ Đức Cảnh) Find the maximum value of the expression

$$f = \frac{a^3 + b^3 + c^3}{abc},$$

where a, b, c are real numbers lying in the interval $[1, 2]$.

276.2 (Hồ Quang Vinh) Given a triangle ABC with sides $BC = a$, $CA = b$, and $AB = c$. Let R and r be the circumradius and inradius of the triangle, respectively. Prove that

$$\frac{a^3 + b^3 + c^3}{abc} \geq 4 - \frac{2r}{R}.$$

276.3 (Phạm Hoàng Hà) Given a triangle ABC, let P be a point on the side BC, let H, K be the orthogonal projections of P onto AB, AC respectively. Points M, N are chosen on AB, AC such that $PM \parallel AC$ and $PN \parallel AB$. Compare the areas of triangles PHK and PMN.

276.4 (Đỗ Thanh Hân) How many 6-digit natural numbers exist with the distinct digits and two arbitrary consecutive digits can not be simultaneously odd numbers?

277.1 (Nguyễn Hối) The incircle with center O of a triangle touches the sides AB, AC, and BC respectively at D, E, and F. The escribed circle of triangle ABC in the angle A has center Q and touches the side BC and the rays AB, AC respectively at K, H, and I. The line DE meets the rays BO and CO respectively at M and N. The line HI meets the rays BQ and CQ at R and S, respectively. Prove that

a) $\triangle FMN = \triangle KRS$, b) $\dfrac{IS}{AB} = \dfrac{SR}{BC} = \dfrac{RH}{CA}$.

277.2 (Nguyễn Đức Huy) Find all rational numbers p, q, r such that

$$p\cos\frac{\pi}{7} + q\cos\frac{2\pi}{7} + r\cos\frac{3\pi}{7} = 1.$$

277.3 (Nguyễn Xuân Hùng) Let $ABCD$ be a bicentric quadrilateral inscribed in a circle with center I and circumcribed about a circle with center O. A line through I, parallel to a side of $ABCD$, intersects its two opposite sides at M and N. Prove that the length of MN does not depend on the choice of side to which the line is parallel.

277.4 (Đinh Thành Trung) Let $x \in (0, \pi)$ be real number and suppose that $\frac{x}{\pi}$ is not rational. Define

$$S_1 = \sin x, \;\; S_2 = \sin x + \sin 2x, \;\; \ldots, \;\; S_n = \sin x + \sin 2x + \cdots + \sin nx.$$

Let t_n be the number of negative terms in the sequence $S_1, S_2, \ldots, S_n$. Prove that $\lim\limits_{n\to\infty} \frac{t_n}{n} = \frac{x}{2\pi}$.

279.1 (Nguyễn Hữu Bằng) Find all natural numbers $a > 1$, such that if p is a prime divisor of a then the number of all divisors of a which are relatively prime to p, is equal to the number of the divisors of a that are not relatively prime to p.

279.2 (Lê Duy Ninh) Prove that for all real numbers a, b, x, y satisfying $x + y = a + b$ and $x^4 + y^4 = a^4 + b^4$ then $x^n + y^n = a^n + b^n$ for all $n \in \mathbb{N}$.

279.3 (Nguyễn Hữu Phước) Given an equilateral triangle ABC, find the locus of points M interior to ABC such that if the

orthogonal projections of M onto BC, CA and AB are D, E, and F, respectively, then AD, BE, and CF are concurrent.

279.4 (Nguyễn Minh Hà) Let M be a point in the interior of triangle ABC and let X, Y, Z be the reflections of M across the sides BC, CA, and AB, respectively. Prove that triangles ABC and XYZ have the same centroid.

279.5 (Vũ Đức Sơn) Find all positive integers n such that $n < t_n$, where t_n is the number of positive divisors of n^2.

279.6 (Trần Nam Dũng) Find the maximum value of the expression

$$\frac{x}{1+x^2} + \frac{y}{1+y^2} + \frac{z}{1+z^2},$$

where x, y, z are real numbers satisfying the condition $x + y + z = 1$.

279.7 (Hoàng Hoa Trại) Given are three concentric circles with center O, and radii $r_1 = 1$, $r_2 = \sqrt{2}$, and $r_3 = \sqrt{5}$. Let A, B, C be three non-collinear points lying respectively on these circles and let F be the area of triangle ABC. Prove that $F \leq 3$, and determine the side lengths of triangle ABC.

281.1 (Nguyễn Xuân Hùng) Let P be a point exterior to a circle with center O. From P construct two tangents touching the circle at A and B. Let Q be a point, distinct from P, on the circle. The tangent at Q of the circle intersects AB and AC at E and F, respectively. Let BC intersect OE and OF at X and Y, respectively. Prove that XY/EF is a constant when P varies on the circle.

281.2 (Hồ Quang Vinh) In a triangle ABC, let $BC = a$, $CA = b$, $AB = c$ be the sides, r, r_a, r_b, and r_c be the inradius and exradii. Prove that

$$\frac{abc}{r} \geq \frac{a^3}{r_a} + \frac{b^3}{r_b} + \frac{c^3}{r_c}.$$

283.1 (Trần Hồng Sơn) Simplify the expression

$$\sqrt{x(4-y)(4-z)} + \sqrt{y(4-z)(4-x)} + \sqrt{z(4-x)(4-y)} - \sqrt{xyz},$$

where x, y, z are positive numbers such that $x + y + z + \sqrt{xyz} = 4$.

283.2 (Nguyễn Phước) Let $ABCD$ be a convex quadrilateral, M be the midpoint of AB. Point P is chosen on the segment AC such that lines MP and BC intersect at T. Suppose that Q is on the segment BD such that $BQ/QD = AP/PC$. Prove that the line TQ has a fixed point when P moves on the segment AC.

284.1 (Nguyễn Hữu Bằng) Given an integer $n > 0$ and a prime $p > n + 1$, prove or disprove that the following equation has integer solutions:

$$1 + \frac{x}{n+1} + \frac{x^2}{2n+1} + \cdots + \frac{x^p}{pn+1} = 0.$$

284.2 (Lê Quang Nẫm) Let x, y be real numbers such that

$$(x + \sqrt{1+y^2})(y + \sqrt{1+x^2}) = 1,$$

prove that

$$(x + \sqrt{1+x^2})(y + \sqrt{1+y^2}) = 1.$$

284.3 (Nguyễn Xuân Hùng) The internal angle bisectors AD, BE, and CF of a triangle ABC meet at point Q. Prove that if the inradii of triangles AQF, BQD, and CQE are equal then triangle ABC is equilateral.

284.4 (Trần Nam Dũng) Disprove that there exists a polynomial $p(x)$ of degree greater than 1 such that if $p(x)$ is an integer then $p(x+1)$ is also an integer for $x \in \mathbb{R}$.

285.1 (Nguyễn Duy Liên) Given an odd natural number p and integers a, b, c, d, e such that $a + b + c + d + e$ and $a^2 + b^2 + c^2 + d^2 + e^2$ are all divisible by p. Prove that $a^5 + b^5 + c^5 + d^5 + e^5 - 5abcde$ is also divisible by p.

285.2 (Vũ Đức Cảnh) Prove that if $x, y \in \mathbb{R}^*$ then

$$\frac{2x^2+3y^2}{2x^3+3y^3} + \frac{2y^2+3x^2}{2y^3+3x^3} \leq \frac{4}{x+y}.$$

285.3 (Nguyễn Hữu Phước) Let P be a point in the interior of triangle ABC. Rays AP, BP, and CP intersect the sides BC, CA, and AB at D, E, and F, respectively. Let K be the point of intersection of DE and CM, H be the point of intersection of DF and BM. Prove that AD, BK and CH are concurrent.

285.4 (Trần Tuấn Anh) Let a, b, c be non-negative real numbers, determine all real numbers x such that the following inequality holds:

$$[a^2 + b^2 + (x-1)c^2][a^2 + c^2 + (x-1)b^2][b^2 + c^2 + (x-1)a^2] \le (a^2 + xbc)(b^2 + xac)(c^2 + xab).$$

285.5 (Trương Cao Dũng) Let O and I be the circumcenter and incenter of a triangle ABC. Rays AI, BI, and CI meet the circumcircle at D, E, and F, respectively. Let R_a, R_b, and R_c be the radii of the escribed circles of $\triangle ABC$, and let R_d, R_e, and R_f be the radii of the escribed circles of triangle DEF. Prove that

$$R_a + R_b + R_c \le R_d + R_e + R_f.$$

285.6 (Đỗ Quang Dương) Determine all integers k such that the sequence defined by $a_1 = 1$, $a_{n+1} = 5a_n + \sqrt{ka_n^2 - 8}$ for $n = 1, 2, 3, \ldots$ includes only integers.

286.1 (Trần Hồng Sơn) Solve the equation

$$18x^2 - 18x\sqrt{x} - 17x - 8\sqrt{x} - 2 = 0.$$

286.2 (Phạm Hùng) Let $ABCD$ be a square. Points E, F are chosen on CB and CD, respectively, such that $BE/BC = k$, and $DF/DC = (1-k)/(1+k)$, where k is a given number, $0 < k < 1$. Segment BD meets AE and AF at H and G, respectively. The line through A, perpendicular to EF, intersects BD at P. Prove that $PG/PH = DG/BH$.

286.3 (Vũ Đình Hòa) In a convex hexagon, the segment joining two of its vertices, dividing the hexagon into two quadrilaterals is called a *principal* diagonal. Prove that in every convex hexagon, in which the length of each side is equal to 1, there exists a principal diagonal with length not greater than 2 and there exists a principal diagonal with length greater than $\sqrt{3}$.

286.4 (Đỗ Bá Chủ) Prove that in any acute or right triangle ABC the following inequality holds:

$$\tan\frac{A}{2} + \tan\frac{B}{2} + \tan\frac{C}{2} + \tan\frac{A}{2}\tan\frac{B}{2}\tan\frac{C}{2} \ge \frac{10\sqrt{3}}{9}.$$

286.5 (Trần Tuấn Điệp) In triangle ABC, no angle exceeding $\frac{\pi}{2}$, and each angle is greater than $\frac{\pi}{4}$. Prove that

$$\cot A + \cot B + \cot C + 3\cot A \cot B \cot C \le 4(2 - \sqrt{2}).$$

287.1 (Trần Nam Dũng) Suppose that a, b are positive integers such that $2a-1$, $2b-1$ and $a+b$ are all primes. Prove that a^b+b^a and a^a+b^b are not divisible by $a+b$.

287.2 (Phạm Đình Trường) Let $ABCD$ be a square in which the two diagonals intersect at E. A line through A meets BC at M and intersects CD at N. Let K be the intersection point of EM and BN. Prove that $CK \perp BN$.

287.3 (Nguyễn Xuân Hùng) Let ABC be a right isosceles triangle, $\angle A = 90°$, I be the incenter of the triangle, M be the midpoint of BC. Let MI intersect AB at N and E be the midpoint of IN. Furthermore, F is chosen on side BC such that $FC = 3FB$. Suppose that the line EF intersects AB and AC at D and K, respectively. Prove that $\triangle ADK$ is isosceles.

287.4 (Hoàng Hoa Trại) Given a positive integer n, and w is the sum of n first integers. Prove that the equation

$$x^3+y^3+z^3+t^3 = 2w^3-1$$

has infinitely many integer solutions.

288.1 (Vũ Đức Cảnh) Find necessary and sufficient conditions for a, b, c for which the following equation has no solutions:

$$a(ax^2+bx+c)^2+b(ax^2+bx+c)+c = x.$$

288.2 (Phạm Ngọc Quang) Let $ABCD$ be a cyclic quadrilateral, P be a variable point on the arc BC not containing A, and F be the foot of the perpendicular from C onto AB. Suppose that $\triangle MEF$ is equilateral, calculate IK/R, where I is the incenter of triangle ABC and K the intersection (distinct from A) of ray AI and the circumcircle of radius R of triangle ABC.

288.3 (Nguyễn Văn Thông) Given a prime $p > 2$ such that $p-2$ is divisible by 3. Prove that the set of integers defined by $y^2 - x^3 - 1$, where x, y are non-negative integers smaller than p, has at most $p-1$ elements divisible by p.

289.1 (Thái Nhật Phượng) Let ABC be a right isosceles triangle with $A = 90°$. Let M be the midpoint of BC, G be a point on side AB such that $GB = 2GA$. Let GM intersect CA at D. The line through M, perpendicular to CG at E, intersects AC at K. Finally, let P be the point of intersection of DE and GK. Prove that $DE = BC$ and $PG = PE$.

289.2 (Hồ Quang Vinh) Given a convex quadrilateral $ABCD$, let M and N be the midpoints of AD and BC, respectively, P be the point of intersection of AN and BM, and Q the intersection point of DN and CM. Prove that

$$\frac{PA}{PN}+\frac{PB}{PM}+\frac{QC}{QM}+\frac{QD}{QN} \geq 4,$$

and determine when equality holds.

290.1 (Nguyễn Song Minh) Given $x, y, z, t \in \mathbb{R}$ and real polynomial

$$F(x,y,z,t) = 9(x^2y^2 + y^2z^2 + z^2t^2 + t^2x^2) + 6xz(y^2 + t^2) - 4xyzt.$$

a) Prove that the polynomial can be factored into the product of two quadratic polynomials.

b) Find the minimum value of the polynomial F if $xy + zt = 1$.

290.2 (Phạm Hoàng Hà) Let M be a point on the internal angle bisector AD of triangle ABC, M distinct from A, D. Ray AM intersects side AC at E, ray CM meets side AB at F. Prove that if

$$\frac{1}{AB^2} + \frac{1}{AE^2} = \frac{1}{AC^2} + \frac{1}{AF^2}$$

then $\triangle ABC$ is isosceles.

290.3 (Đỗ Ánh) Consider a triangle ABC and its incircle. The internal angle bisector AD and median AM intersect the incircle again at P and Q, respectively. Compare the lengths of DP and MQ.

290.4 (Nguyễn Duy Liên) Find all pairs of integers (a, b) such that $a + b^2$ divides $a^2b - 1$.

290.5 (Đinh Thành Trung) Determine all real functions $f(x), g(x)$ such that $f(x) - f(y) = \cos(x+y) \cdot g(x-y)$ for all $x, y \in \mathbb{R}$.

290.6 (Nguyễn Minh Đức) Find all real numbers a such that the system of equations has real solutions in x, y, z:

$$\sqrt{x-1} + \sqrt{y-1} + \sqrt{z-1} = a - 1,$$
$$\sqrt{x+1} + \sqrt{y+1} + \sqrt{z+1} = a + 1.$$

290.7 (Đoàn Kim Sang) Given a positive integer n, find the number of positive integers, not exceeding $n(n+1)(n+2)$, which are divisible by n, $n+1$, and $n+2$.

291.1 (Bùi Minh Duy) Given three distinct numbers a, b, c such that

$$\frac{a}{b-c} + \frac{b}{c-a} + \frac{c}{a-b} = 0,$$

prove that any two of the numbers have different signs.

291.2 (Đỗ Thanh Hân) Given three real numbers x, y, z that satisfy the conditions $0 < x < y \leq z \leq 1$ and $3x + 2y + z \leq 4$. Find the maximum value of the expression $3x^3 + 2y^2 + z^2$.

291.3 (Vi Quốc Dũng) Given a circle of center O and two points A, B on the circle. A variable circle through A, B has center Q. Let P be the reflection of Q across the line AB. Line AP intersects the circle O again at E, while line BE, E distinct from B, intersects the circle Q again at F. Prove that F lies on a fixed line when circle Q varies.

291.4 (Vũ Đức Sơn) Find all functions $f : \mathbb{Q} \to \mathbb{Q}$ such that

$$f(f(x) + y) = x + f(y) \quad \text{for } x, y \in \mathbb{Q}.$$

291.5 (Nguyễn Văn Thông) Find the maximum value of the expression

$$x^2(y - z) + y^2(z - y) + z^2(1 - z),$$

where x, y, z are real numbers such that $0 \leq x \leq y \leq z \leq 1$.

291.6 (Vũ Thành Long) Given an acute-angled triangle ABC with side lengths a, b, c. Let R, r denote its circumradius and inradius, respectively, and F its area. Prove the inequality

$$ab + bc + ca \geq 2R^2 + 2Rr + \frac{8}{\sqrt{3}}F.$$

292.1 (Thái Nhật Phượng, Trần Hà) Let x, y, z be positive numbers such that $xyz = 1$, prove the inequality

$$\frac{x^2}{x + y + y^3 z} + \frac{y^2}{y + z + z^3 x} + \frac{z^2}{z + x + x^3 y} \leq 1.$$

292.2 (Phạm Ngọc Bội) Let p be an odd prime, let $a_1, a_2, \ldots, a_{p-1}$ be $p - 1$ integers that are not divisible by p. Prove that among the sums $T = k_1 a_1 + k_2 a_2 + \cdots + k_{p-1} a_{p-1}$, where $k_i \in \{-1, 1\}$ for $i = 1, 2, \ldots, p - 1$, there exists at least a sum T divisible by p.

292.3 (Ha Vu Anh) Given are two circles Γ_1 and Γ_2 intersecting at two distinct points A, B and a variable point P on Γ_1, P distinct from A and B. The lines PA, PB intersect Γ_2 at D and E, respectively. Let M be the midpoint of DE. Prove that the line MP has a fixed point.

295.1 (Hoàng Văn Đắc) Let $a, b, c, d \in \mathbb{R}$ such that $a + b + c + d = 1$, prove that

$$(a + c)(b + d) + 2(ac + bd) \leq \frac{1}{2}.$$

294.1 (Phùng Trọng Thực) Triangle ABC is inscribed in a circle of center O. Let M be a point on side AC, M distinct from A, C, the line BM meets the circle again at N. Let Q be the intersection of a line through A perpendicular to AB and a line through N perpendicular to NC. Prove that the line QM has a fixed point when M varies on AC.

294.2 (Trần Xuân Bang) Let A, B be the intersections of circle O of radius R and circle O' of radius R'. A line touches circle O and O' at T and T', respectively. Prove that B is the centroid of triangle ATT' if and only if

$$OO' = \frac{\sqrt{3}}{2}(R + R').$$

294.3 (Vũ Trí Đức) If a, b, c are positive real numbers such that $ab + bc + ca = 1$, find the minimum value of the expression $w(a^2 + b^2) + c^2$, where w is a positive real number.

294.4 (Lê Quang Nẫm) Let p be a prime greater than 3, prove that $\binom{p-1}{2001p^2-1} - 1$ is divisible by p^4.

294.5 (Trương Ngọc Đắc) Let x, y, z be positive real numbers such that $x = \max\{x, y, z\}$, find the minimum value of

$$\frac{x}{y} + \sqrt{1 + \frac{y}{z}} + \sqrt[3]{1 + \frac{z}{x}}.$$

294.6 (Phạm Hoàng Hà) The sequence (a_n), $n = 1, 2, 3, \ldots$, is defined by $a_n = \frac{1}{n^2(n+2)\sqrt{n+1}}$ for $n = 1, 2, 3, \ldots$. Prove that

$$a_1 + a_2 + \cdots + a_n < \frac{1}{2\sqrt{2}} \qquad \text{for} \quad n = 1, 2, 3, \ldots.$$

294.7 (Vũ Huy Hoàng) Given are a circle O of radius R, and an odd natural number n. Find the positions of n points $A_1, A_2, \ldots, A_n$ on the circle such that the sum $A_1A_2 + A_2A_3 + \cdots + A_{n-1}A_n + A_nA_1$ is a minimum.

295.2 (Trần Tuyết Thanh) Solve the equation

$$x^2 - x - 1000\sqrt{1 + 8000x} = 1000.$$

295.3 (Phạm Đình Trường) Let $A_1A_2A_3A_4A_5A_6$ be a convex hexagon with parallel opposite sides. Let B_1, B_2, and B_3 be the points of intersection of pairs of diagonals A_1A_4 and A_2A_5, A_2A_5 and A_3A_6, A_3A_6 and A_1A_4, respectively. Let C_1, C_2, C_3 be respectively the midpoints of the segments A_3A_6, A_1A_4, A_2A_5. Prove that B_1C_1, B_2C_2, B_3C_3 are concurrent.

295.4 (Bùi Thế Hùng) Let A, B be respectively the greatest and smallest numbers from the set of n positive numbers $x_1, x_2, \ldots, x_n$, $n \geq 2$. Prove that

$$A < \frac{(x_1 + x_2 + \cdots + x_n)^2}{x_1 + 2x_2 + \cdots + nx_n} < 2B.$$

295.5 (Trần Tuấn Anh) Prove that if $x, y, z > 0$ then

a) $(x+y+z)^3(y+z-x)(z+x-y)(x+y-z) \leq 27x^3y^3z^3$,

b) $(x^2+y^2+z^2)(y+z-x)(z+x-y)(x+y-z) \leq xyz(yz+zx+xy)$,

c) $(x+y+z)\left[2(yz+zx+xy)-(x^2+y^2+z^2)\right] \leq 9xyz$.

295.6 (Vũ Thị Huệ Phương) Find all functions $f : \mathbb{D} \to \mathbb{D}$, where $\mathbb{D} = [1, +\infty)$ such that

$$f(xf(y)) = yf(x) \quad \text{for } x, y \in \mathbb{D}.$$

295.7 (Nguyễn Viết Long) Given an even natural number n, find all polynomials $p_n(x)$ of degree n such that

i) all the coefficients of $p_n(x)$ are elements from the set $\{0, -1, 1\}$ and $p_n(0) \neq 0$;

ii) there exists a polynomial $q(x)$ with coefficients from the set $\{0, -1, 1\}$ such that $p_n(x) \equiv (x^2-1)q(x)$.

296.1 (Thới Ngọc Anh) Prove that

$$\frac{1}{6} < \frac{3 - \underbrace{\sqrt{6+\sqrt{6+\cdots+\sqrt{6}}}}_{n \text{ times}}}{3 - \underbrace{\sqrt{6+\sqrt{6+\cdots+\sqrt{6}}}}_{(n-1) \text{ times}}} < \frac{5}{27},$$

where there are n radical signs in the expression of the numerator and $n-1$ ones in the expression of the denominator.

296.2 (Vi Quốc Dũng) Let ABC be a triangle and M the midpoint of BC. The external angle bisector of A meets BC at D. The circumcircle of triangle ADM intersects line AB and line AC at E and F, respectively. If N is the midpoint of EF, prove that $MN \parallel AD$.

296.3 (Nguyễn Văn Hiến) Let $k, n \in \mathbb{N}$ such that $k < n$. Prove that

$$\frac{(n+1)^{n+1}}{(k+1)^{k+1}(n-k+1)^{n-k+1}} < \frac{n!}{k!\,(n-k)!} < \frac{n^n}{k^k(n-k+1)^{n-k}}.$$

297.1 (Nguyễn Hữu Phước) Given a circle with center O and diameter EF. Points N, P are chosen on line EF such that $ON = OP$. From a point M interior to the circle, not lying on EF, draw MN intersecting the circle at A and C, draw MP meeting the circle at B and D such that B and O are on different sides of AC. Let K be the point of intersection of OB and AC, Q the point of intersection of EF and CD. Prove that lines KQ, BD, AO are concurrent.

297.2 (Trần Nam Dũng) Let a and b two relatively prime numbers. Prove that there exist exactly $\frac{1}{2}(ab - a - b + 1)$ natural numbers that can not be written in the form $ax + by$, where x and y are non-negative integers.

297.3 (Lê Quốc Hán) The circle with center I and radius r touches the sides $BC = a$, $CA = b$, and $AB = c$ of triangle ABC at M, N, and P, respectively. Let F be the area of triangle ABC and h_a, h_b, h_c be the lengths of the altitudes of $\triangle ABC$. Prove that

a) $4F^2 = ab \cdot MN^2 + bc \cdot NP^2 + ca \cdot PM^2$;

b) $\dfrac{MN^2}{h_a h_b} + \dfrac{NP^2}{h_b h_c} + \dfrac{PM^2}{h_c h_a} = 1.$

298.1 (Phạm Hoàng Hà) Let P be the midpoint of side BC of triangle ABC and let BE, CF be two altitudes of the triangle. The line through A, perpendicular to PF, meets CF at M; the line through A, perpendicular to PE, intersects BE at N. Let K and G be respectively the midpoints of BM and CN. Finally, let H be the intersection of KF and GE. Prove that AH is perpendicular to EF.

298.2 (Phạm Đình Trường) Let $ABCD$ be a square. Points E and F are chosen on sides AB and CD, respectively, such that $AE = CF$. Let AD intersect CE and BF at M and N, respectively. Suppose that P is the intersection of BM and CN, find the locus of P when E and F move on the side AB and CD, respectively.

298.3 (Nguyễn Minh Hà) Let $ABCD$ be a convex quadrilateral, let AB intersect CD at E; AD intersects BC at F. Prove that the midpoints of line segments AB, CD, and EF are collinear.

298.4 (Nguyễn Minh Hà) Given a cylic quadrilateral $ABCD$, M is any point in the plane. Let X, Y, Z, T, U, V be the orthogonal projections of M on the lines AB, CD, AC, DB, AD, and BC. Let E, F, G be the midpoints of XY, ZT, and UV. Prove that E, F, and G are collinear.

300.1 (Vũ Trí Đức) Find the maximum and minimum values of the expression $x\sqrt{1+y} + y\sqrt{1+x}$, where x, y are non-negative real numbers such that $x + y = 1$.

300.2 (Nguyễn Xuân Hùng) Let P be a point in the interior of triangle ABC. The incircle of triangle ABC is tangent to sides BC, CA and AB at D, E, and F, respectively. The incircle of triangle PBC touches the sides BC, CP, and PB at K, M, and N, respectively. Suppose that Q is the point of intersection of lines EM and FN. Prove that A, P, Q are collinear if and only if K coincides with D.

300.3 (Huỳnh Tấn Châu) Determine all pairs of integers (m, n) such that

$$\frac{n}{m} = \frac{(m^2 - n^2)^{n/m} - 1}{(m^2 - n^2)^{n/m} + 1}.$$

300.4 (Võ Giang Giai, Mạnh Tú) Prove that if $a, b, c, d, e \geq 0$ then

$$\frac{a+b+c+d+e}{5} \geq \sqrt[5]{abcde} + \frac{q}{20},$$

where $q = (\sqrt{a} - \sqrt{b})^2 + (\sqrt{b} - \sqrt{c})^2 + (\sqrt{c} - \sqrt{d})^2 + (\sqrt{d} - \sqrt{e})^2$.

301.1 (Lê Quang Nẫm) Find all pairs of integers (x, y) such that $x^2 + xy + y^2 + 14x + 14y + 2018$ is divisible by 101.

301.2 (Nguyễn Thế Bình) Find smallest value of the expression

$$\frac{2}{ab} + \frac{1}{a^2+b^2} + \frac{a^4+b^4}{2}$$

where a, b are real positive numbers such that $a + b = 1$.

301.3 (Đỗ Anh) Suppose that a, b, c are side lengths of a triangle and $0 \leq t \leq 1$. Prove that

$$\sqrt{\frac{a}{b+c-ta}} + \sqrt{\frac{b}{c+a-tb}} + \sqrt{\frac{c}{a+b-tc}} \geq 2\sqrt{1+t}.$$

301.4 (Nguyễn Trọng Tuấn) The sequence (a_n) is defined by $a_1 = 5, a_2 = 11$ and $a_{n+1} = 2a_n - 3a_{n-1}$ for $n = 2, 3, \ldots$ Prove that the sequence has indefinitely many positve and negative terms, and show that a_{2002} is divisible by 11.

301.5 (Trần Xuân Đáng) Find the maximum value of $3(a+b+c) - 22abc$, where $a, b, c \in \mathbb{R}$ such that $a^2 + b^2 + c^2 = 1$.

301.6 (Nguyễn Văn Tình) Given is an equilateral triangle ABC with centroid G. A variable line through the centroid and intersects the side BC, CA, and AB at M, N, and P respectively. Prove that $GM^{-4} + GN^{-4} + GP^{-4}$ is a constant.

301.7 (Lê Hào) A convex quadrilateral $ABCD$ is inscribed in a circle with center O, radius R. Let CD intersect AB at E, a line through E meets the lines AD and BC at P, Q. Prove that

$$\frac{1}{EP} + \frac{1}{EQ} \leq \frac{2EO}{EO^2 - R^2},$$

and determine when equality holds.

306.1 (Phan Thị Mùi) Prove that if $x, y, z > 0$ and $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$ then

$$(x + y - z - 1)(y + z - x - 1)(z + x - y - 1) \leq 8.$$

306.2 (Trần Tuấn Anh) Given an integer $m \geq 4$, find the maximum and minimum values of the expression $ab^{m-1} + a^{m-1}b$, where a, b are real numbers such that $a + b = 1$ and $0 \leq a, b \leq \frac{m-2}{m}$.

308.1 (Lê Thị Anh Thư) Find all integer solutions of the equation

$$4(a - x)(x - b) + b - a = y^2,$$

where a, b are given integers, $a > b$.

308.2 (Phan Thế Hải) Given a convex quadrilateral $ABCD$, E is the point of intersection of AB and CD, and F is the intersection of AD and BC. The diagonals AC and BD meet at O. Suppose that M, N, P, Q are the midpoints of AB, BC, CD, and DA. Let H be the intersection of OF and MP, and K the intersection of OE and NQ. Prove that $HK \parallel EF$.

309.1 (Vũ Hoàng Hiệp) Given a positive integer n, find the smallest possible $t = t(n)$ such that for all real numbers $x_1, x_2, \ldots, x_n$ we have

$$\sum_{k=1}^{n} (x_1 + x_2 + \cdots + x_k)^2 \leq t(x_1^2 + x_2^2 + \cdots + x_n^2).$$

309.2 (Lê Xuân Sơn) Given a triangle ABC, prove that

$$\sin A \cos B + \sin B \cos C + \sin C \cos A \leq \frac{3\sqrt{3}}{4}.$$

311.1 (Nguyễn Xuân Hùng) The chord PQ of the circumcircle of a triangle ABC meets its incircle at M and N. Prove that $PQ \geq 2MN$.

311.2 (Đàm Văn Nhỉ) Given a convex quadrilateral $ABCD$ with perpendicular diagonals AC and BD, let BC intersect AD at I and let AB meet CD at J. Prove that $BDIJ$ is cyclic if and only if $AB \cdot CD = AD \cdot BC$.

318.1 (Đậu Thị Hoàng Oanh) Prove that if $2n$ is a sum of two distinct perfect square numbers (greater than 1) then n^2+2n is the sum of four perfect square numbers (greater than 1).

318.2 (Nguyễn Đễ) Solve the system of equations

$$x^2(y+z)^2 = (3x^2+x+1)y^2z^2,$$
$$y^2(z+x)^2 = (4y^2+y+1)z^2x^2,$$
$$z^2(x+y)^2 = (5z^2+z+1)x^2y^2.$$

318.3 (Trần Việt Hùng) A quadrilateral $ABCD$ is insribed in a circle such that the circle of diameter CD intersects the line segments AC, AD, BC, BD respectively at A_1, A_2, B_1, B_2, and the circle of diameter AB meets the line segments CA, CB, DA, DB respectively at C_1, C_2, D_1, D_2. Prove that there exists a circle that is tangent to the four lines A_1A_2, B_1B_2, C_1C_2 and D_1D_2.

319.1 (Dương Châu Dinh) Prove the inequality

$$x^2y+y^2z+z^2x \le x^3+y^3+z^3 \le 1+\frac{1}{2}(x^4+y^4+z^4),$$

where x, y, z are real non-negative numbers such that $x+y+z=2$.

319.2 (Tô Minh Hoàng) Find all functions $f:\mathbb{N}\to\mathbb{N}$ such that

$$2(f(m^2+n^2))^3 = f^2(m)f(n)+f^2(n)f(m)$$

for distinct m and n.

319.3 (Trần Việt Anh) Suppose that AD, BE and CF are the altitudes of an acute triangle ABC. Let M, N, and P be the intersection points of AD and EF, BE and FD, CF and DE respectively. Denote the area of triangle XYZ by $F[XYZ]$. Prove that

$$\frac{1}{F[ABC]} \le \frac{F[MNP]}{F^2[DEF]} \le \frac{1}{8\cos A\cos B\cos C\cdot F[ABC]}.$$

320.1 (Nguyễn Quang Long) Find the maximum value of the function $f=\sqrt{4x-x^3}+\sqrt{x+x^3}$ for $0\le x\le 2$.

320.2 (Vũ Đĩnh Hòa) Two circles of centers O and O' intersect at P and Q (see Figure). The common tangent, adjacent to P, of the two circles touches O at A and O' at B. The tangent of circle O at P intersects O' at C; and the tangent of O' at P meets the circle O at D. Let M be the reflection of P across the midpoint of AB. The line AP intersects BC at E and the line BP meets AD at F. Prove that the hexagon $AMBEQF$ is cyclic.

320.3 (Hồ Quang Vinh) Let R and r be the circumradius and inradius of triangle ABC; the incircle touches the sides of the triangle at three points which form a triangle of perimeter p. Suppose that q is the perimeter of triangle ABC. Prove that $r/R \leq p/q \leq \frac{1}{2}$.

321.1 (Lê Thanh Hải) Prove that for all positive numbers a, b, c, d

$$\text{a)}\ \frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq \frac{a+b+c}{\sqrt[3]{abc}};$$

$$\text{b)}\ \frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{d^2} + \frac{d^2}{a^2} \geq \frac{a+b+c+d}{\sqrt[4]{abcd}}.$$

321.2 (Phạm Hoàng Hà) Find necessary and sufficient conditions for which the system of equations

$$\begin{aligned} x^2 &= (2+m)y^3 - 3y^2 + my, \\ y^2 &= (2+m)z^3 - 3z^2 + mz, \\ z^2 &= (2+m)x^3 - 3x^2 + mx \end{aligned}$$

has a unique solution.

321.3 (Trần Việt Anh) Let m, n, p be three positive integers such that $n+1$ is divisible by m. Find a formula for the set of numbers $(x_1, x_2, \ldots, x_p)$ of p positive primes such that the sum $x_1 + x_2 + \cdots + x_p$ is divisible by m, with each number of the set not exceeding n.

322.1 (Nguyễn Như Hiền) Given a triangle ABC with incenter I. The lines AI and DI intersect the circumcircle of triangle ABC again at H and K, respectively. Draw IJ perpendicular to BC at J. Prove that H, K and J are collinear.

322.2 (Trần Tuấn Anh) Prove the inequality

$$\frac{1}{2}\left(\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \frac{1}{x_i}\right) \geq n - 1 + \frac{n}{\sum_{i=1}^{n} x_i},$$

where x_i $(i = 1, 2, \ldots, n)$ are positive real numbers such that $\sum_{i=1}^{n} x_i^2 = n$, with n as an integer, $n > 1$.

323.1 (Nguyễn Đức Thuận) Suppose that $ABCD$ is a convex quadrilateral. Points E, F are chosen on the lines BC and AD, respectively, such that $AE \parallel CD$ and $CF \parallel AB$. Prove that A, B, C, D are concyclic if and only if $AECF$ has an incircle.

323.2 (Nguyễn Thế Phiệt) Prove that for an acute triangle ABC,

$$\cos A + \cos B + \cos C + \frac{1}{3}(\cos 3B + \cos 3C) \geq \frac{5}{6}.$$

324.1 (Trần Nam Dũng) Find the greatest possible real number c such that we can always choose a real number x which satisfies the inequality $\sin(mx) + \sin(nx) \geq c$ for each pair of positive integers m and n.

325.1 (Nguyễn Đăng Phất) Given a convex hexagon inscribed in a circle such that the opposite sides are parallel. Prove that the sums of the lengths of each pair of opposite sides are equal if and only if the distances of the opposite sides are the same.

325.2 (Đinh Văn Khâm) Given a natural number n and a prime p, how many sets of p natural numbers $\{a_0, a_1, \ldots, a_{p-1}\}$ are there such that

a) $1 \leq a_i \leq n$ for each $i = 0, 1, \ldots, p-1$,

b) $[a_0, a_1, \ldots, a_{p-1}] = p \min\{a_0, a_1, \ldots, a_{p-1}\}$,

where $[a_0, a_1, \ldots, a_{p-1}]$ denotes the least common multiple of the numbers $a_0, a_1, \ldots, a_{p-1}$?

327.1 (Hoàng Trọng Hảo) Let $ABCD$ be a bicentric quadrilateral (i.e., it has a circumcircle of radius R and an incircle of radius r). Prove that $R \geq r\sqrt{2}$.

327.2 (Vũ Đình Thế) Two sequences (x_n) and (y_n) are defined by

$$x_{n+1} = -2x_n^2 - 2x_n y_n + 8y_n^2, \quad x_1 = -1,$$
$$y_{n+1} = 2x_n^2 + 3x_n y_n - 2y_n^2, \quad y_1 = 1$$

for $n = 1, 2, 3, \ldots$. Find all primes p such that $x_p + y_p$ is not divisible by p.

328.1 (Bùi Văn Chi) Find all integer solutions (n, m) of the equation

$$(n+1)(2n+1) = 10m^2.$$

328.2 (Nguyễn Thị Minh) Determine all positive integers n such that the polynomial of $n+1$ terms

$$p(x) = x^{4n} + x^{4(n-1)} + \cdots + x^8 + x^4 + 1$$

is divisible by the polynomial of $n+1$ terms

$$q(x) = x^{2x} + x^{2(n-1)} + \cdots + x^4 + x^2 + 1.$$

328.3 (Bùi Thế Hùng) Find the smallest possible prime p such that $[(3+\sqrt{p})^{2n}]+1$ is divisible by 2^{n+1} for each natural number n, where $[x]$ denotes the integral part of x.

328.4 (Hàn Ngọc Đức) Find all real numbers a such that there exists a positive real number k and functions $f:\mathbb{R}\to\mathbb{R}$ which satisfy the inequality

$$\frac{f(x)+f(y)}{2} \geq f\left(\frac{x+y}{2}\right)+k|x-y|^a,$$

for all real numbers x, y.

328.5 (Vũ Hoàng Hiệp) In space, let $A_1, A_2, \ldots, A_n$ be n distinct points. Prove that

$$\text{a)}\quad \sum_{i=1}^{n} \angle A_iA_{i+1}A_{i+2} \geq \pi,$$

$$\text{b)}\quad \sum_{i=1}^{n} \angle A_iQA_{i+1} \leq (n-1)\pi,$$

where A_{n+i} is equal to A_i and Q is an arbitrary point distinct from $A_1, A_2, \ldots, A_n$.

329.1 (Hoàng Ngọc Minh) Find the maximum value of the expression

$$(a-b)^4+(b-c)^4+(c-a)^4,$$

for any real numbers $1 \leq a, b, c \leq 2$.

331.1 (Nguyễn Mạnh Tuấn) Let x, y, z, w be rational numbers such that $x+y+z+w=0$. Show that the number

$$\sqrt{(xy-zw)(yz-wx)(zx-yw)}$$

is also rational.

331.2 (Bùi Đình Thân) Given positive reals a, b, c, x, y, z such that

$$a+b+c=4 \quad \text{and} \quad ax+by+cz=xyz,$$

show that $x+y+z>4$.

331.3 (Phạm Năng Khánh) Given a triangle ABC and its angle bisector AM, the line perpendicular to BC at M intersects line AB at N. Prove that $\angle BAC$ is a right angle if and only if $MN = MC$.

331.4 (Đào Tam) Diagonals AC, BD of quadrilateral $ABCD$ intersect at I such that $IA = ID$ and $\angle AID = 120^\circ$. From point M on segment BC, draw $MN \parallel AC$ and $MQ \parallel BD$, N and Q are on AB and CD, respectively. Find the locus of circumcenter of triangle MNQ when M moves on line segment BC.

331.5 (Nguyễn Trọng Hiệp) Let p, q be primes such that $p > q > 2$. Find all integers k such that the equation $(px - qy)^2 = kxyz$ has integer solutions (x, y, z) with $xy \neq 0$.

331.6 (Hàn Ngọc Đức) Let a sequence (u_n), $n = 1, 2, 3, \ldots$, be given defined by $u_n = n^{2^n}$ for all $n = 1, 2, \ldots$. Let

$$x_n = \frac{1}{u_1} + \frac{1}{u_2} + \cdots + \frac{1}{u_n}.$$

Prove that the sequence (x_n) has a limit as n tends to infinity and that the limit is irrational.

331.7 (Trần Tuấn Anh) Find all positive integers $n \geq 3$ such that the following inequality holds for all real numbers $a_1, a_2, \ldots, a_n$ (assume $a_{n+1} = a_1$)

$$\sum_{1 \leq i < j \leq n} (a_i - a_j)^2 \leq \left(\sum_{i=1}^{n} |a_i - a_{i+1}| \right)^2 .$$

332.1 (Nguyễn Văn Ái) Find the remainder in the integer division of the number $a^b + b^a$ by 5, where $a = \overline{22\ldots2}$ with 2004 digits 2, and $b = \overline{33\ldots3}$ with 2005 digits 3 (written in the decimal system).

332.2 (Nguyên Khánh Nguyên) Suppose that ABC is an isosceles triangle with $AB = AC$. On the line perpendicular to AC at C, let point D such that points B, D are on different sides of AC. Let K be the intersection point of the line perpendicular to AB at B and the line passing through the midpoint M of CD, perpendicular to AD. Compare the lengths of KB and KD.

332.3 (Phạm Văn Hoàng) Consider the equation

$$x^2 - 2kxy^2 + k(y^3 - 1) = 0,$$

where k is some integer. Prove that the equation has integer solutions (x, y) such that $x > 0$, $y > 0$ if and only if k is a perfect square.

332.4 (Đỗ Văn Ta) Solve the equation

$$\sqrt{x - \sqrt{x - \sqrt{x - \sqrt{x - 5}}}} = 5.$$

332.5 (Phạm Xuân Trinh) Show that if $a \geq 0$ then

$$\sqrt{a} + \sqrt[3]{a} + \sqrt[6]{a} \leq a + 2.$$

332.6 (Bùi Văn Chi) Let $ABCD$ be a parallelogram with $AB < BC$. The bisector of angle $\angle BAD$ intersects BC at E; let O be the intersection point of the perpendicular bisectors of BD and CE. A line passing through C parallel to BD intersects the circle with center O and radius OC at F. Determine $\angle AFC$.

332.7 (Phan Hoàng Ninh) Prove that the polynomial

$$p(x) = x^4 - 2003x^3 + (2004 + a)x^2 - 2005x + a$$

with $a \in \mathbb{Z}$ has at most one integer solution. Furthermore, prove that it has no multiple integral root greater than 1.

332.8 (Phùng Văn Sử) Prove that for any real numbers a, b, c

$$(a^2 + 3)(b^2 + 3)(c^2 + 3) \geq \frac{4}{27}(3ab + 3bc + 3ca + abc)^2.$$

332.9 (Nguyễn Văn Thành) Determine all functions $f(x)$ defined on the interval $(0, +\infty)$ which have a derivative at $x = 1$ and that satisfy

$$f(xy) = \sqrt{x}f(y) + \sqrt{y}f(x)$$

for all positive real numbers x, y.

332.10 (Hoàng Ngọc Cảnh) Let $A_1A_2 \ldots A_n$ be a n-gon inscribed in the unit circle; let M be a point on the minor arc A_1A_n. Prove that

a) $MA_1 + MA_3 + \cdots + MA_{n-2} + MA_n < \dfrac{n}{\sqrt{2}}$ for n odd;

b) $MA_1 + MA_3 + \cdots + MA_{n-3} + MA_{n-1} \leq \dfrac{n}{\sqrt{2}}$ for n even.

When does equality hold?

332.11 (Đặng Thanh Hải) Let ABC be an equilateral triangle with centroid O; ℓ is a line perpendicular to the plane (ABC) at O. For each point S on ℓ, distinct from O, a pyramid $SABC$ is defined. Let ϕ be the dihedral angle between a lateral face and the base, let *ma* be the angle between two adjacent lateral faces of the pyramid. Prove that the quantity $F(\phi, \gamma) = \tan^2 \phi \, [3 \tan^2(\gamma/2) - 1]$ is independent of the position of S on ℓ.

334.1 (Đặng Như Tuấn) Determine the sum

$$\frac{1}{1 \cdot 2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \cdots + \frac{1}{(n-1)n(n+1)} + \cdots + \frac{1}{23 \cdot 24 \cdot 25}.$$

334.2 (Nguyễn Phước) Let ABC be a triangle with angle A not being right, $B \neq 135^\circ$. Let M be the midpoint of BC. A right isosceles triangle ABD is outwardly erected on the side BC as base. Let E be the intersection point of the line through A perpendicular to AB and the line through C parallel to MD. Let AB intersect CE and DM at P and Q, respectively. Prove that Q is the midpoint of BP.

334.3 (Nguyễn Duy Liên) Find the smallest possible odd natural number n such that n^2 can be expressed as the sum of an odd number of consecutive perfect squares.

334.4 (Phạm Việt Hải) Find all positive numbers a, b, c, d such that

$$\frac{a^2}{b+c} + \frac{b^2}{c+d} + \frac{c^2}{d+a} + \frac{d^2}{a+b} = 1 \quad \text{and}$$
$$a^2 + b^2 + c^2 + d^2 \geq 1.$$

334.5 (Đào Quốc Dũng) The incircle of triangle ABC (incenter I) touches the sides BC, CA, and AB respectively at D, E, F. The line through A perpendicular to IA intersects lines DE, DF at M, N, respectively; the line through B perpendicular to IB intersect EF, ED at P, Q, respectively; the line through C perpendicular to IC intersect lines FD, FE at S, T, respectively. Prove the inequality

$$MN + PQ + ST \geq AB + BC + CA.$$

334.6 (Vũ Hữu Bình) Let ABC be a right isosceles triangle with $A = 90^\circ$. Find the locus of points M such that $MB^2 - MC^2 = 2MA^2$.

334.7 (Trần Tuấn Anh) We are given n distinct positive numbers, $n \geq 4$. Prove that it is possible to choose at least two numbers such that their sums and differences do not coincide with any $n-2$ others of the given numbers.

335.1 (Vũ Tiến Việt) Prove that for all triangles ABC

$$\cos A + \cos B + \cos C \leq 1 + \frac{1}{6}\left(\cos^2\frac{A-B}{2} + \cos^2\frac{B-C}{2} + \cos^2\frac{C-A}{2}\right).$$

335.2 (Phan Đức Tuấn) In triangle ABC, let $BC = a$, $CA = b$, $AB = c$ and F be its area. Suppose that M, N, and P are points on
the sides BC, CA, and AB, respectively. Prove that

$$ab \cdot MN^2 + bc \cdot NP^2 + ca \cdot PM^2 \geq 4F^2.$$

335.3 (Trần Văn Xuân) In isosceles triangle ABC, $\angle ABC = 120^\circ$. Let D be the point of intersection of line BC and the tangent to the circumcircle of triangle ABC at A. A line through D and the circumcenter O intersects AB and AC at E and F, respectively. Let M and N be the midpoints of AB and AC. Show that AO, MF and NE are concurrent.

336.1 (Nguyễn Hòa) Solve the following system of equations

$$\frac{a}{x} - \frac{b}{z} = c - zx,$$
$$\frac{b}{y} - \frac{c}{x} = a - xy,$$
$$\frac{c}{z} - \frac{a}{y} = b - yz.$$

336.2 (Phạm Văn Thuận) Given two positive real numbers a, b such that $a^2 + b^2 = 1$, prove that

$$\frac{1}{a} + \frac{1}{b} \geq 2\sqrt{2} + \left(\sqrt{\frac{a}{b}} - \sqrt{\frac{b}{a}}\right)^2.$$

336.3 (Nguyễn Hồng Thanh) Let P be an arbitrary point in the interior of triangle ABC. Let $BC = a$, $CA = b$, $AB = c$. Denote by u, v and w the distances of P to the lines BC, CA, AB, respectively. Determine P such that the product uvw is a maximum and calculate this maximum in terms of a, b, c.

336.4 (Nguyễn Lâm Tuyền) Given the polynomial $Q(x) = (p-1)x^p - x - 1$ with p being an odd prime number. Prove that there exist infinitely many positive integers a such that $Q(a)$ is divisible by p^p.

336.5 (Hoàng Minh Dũng) Prove that in any triangle ABC the following inequalities hold:

a) $\cos A + \cos B + \cos C + \cot A + \cot B + \cot C \geq \frac{3}{2} + \sqrt{3}$;

b) $\sqrt{3}(\cos A + \cos B + \cos C) + \cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} \geq \frac{9\sqrt{3}}{2}$.

337.1 (Nguyễn Thị Loan) Given four real numbers a, b, c, d such that $4a^2 + b^2 = 2$ and $c + d = 4$, determine the maximum value of the expression $f = 2ac + bd + cd$.

337.2 (Vũ Anh Nam) In triangle ABC, let D be the intersection point of the internal angle bisectors BM and CN, M on AC and N on AB. Prove that $\angle BAC = 90°$ if and only if $2BD \cdot CD = BM \cdot CN$.

337.3 (Trần Tuấn Anh) Determine the maximum value of the expression $f = (x-y)(y-z)(z-x)(x+y+z)$, where x, y, z lie in the interval $[0, 1]$.

337.4 (Hàn Ngọc Đức) Let n, $n \geq 2$, be a natural number, a, b be positive real numbers such that $a < b$. Suppose that $x_1, x_2, \ldots, x_n$ are n real numbers in the interval $[a, b]$. Find the maximum value of the sum

$$\sum_{1 \leq i < j \leq n} (x_i - x_j)^2.$$

337.5 (Lê Hoài Bắc) A line through the incenter of a triangle ABC intersects sides AB and AC at M and N, respectively. Show that

$$\frac{MB \cdot NC}{MA \cdot NA} \leq \frac{BC^2}{4\,AB \cdot AC}.$$

338.1 (Phạm Thịnh) Show that if a, b, c, d, p, q are positive real numbers with $p \geq q$ then the following inequality holds:

$$\frac{a}{pb + qc} + \frac{b}{pc + qd} + \frac{c}{pd + qa} + \frac{d}{pa + qb} \geq \frac{4}{p + q}.$$

Is the inequality still true if $p < q$?

338.2 (Trần Quang Vinh) Determine all functions $f : \mathbb{R} \to \mathbb{R}$ satisfying the condition $f(x^2 + f(y)) = y + xf(x)$ for all real numbers x, y.

338.3 (Trần Việt Anh) Determine the smallest possible positive integer n such that there exists a polynomial $p(x)$ of degree n with integer coefficients satisfying the conditions

a) $p(0) = 1,\ p(1) = 1;$

b) $p(m)$ divided by 2003 leaves remainders 0 or 1 for all integers $m > 0$.

338.4 (Hoàng Trọng Hảo) The Fibonacci sequence (F_n), $n = 1, 2, \ldots$, is defined by $F_1 = F_2 = 1$, $F_{n+1} = F_n + F_{n-1}$ for $n = 2, 3, 4, \ldots$. Show that if $a \neq F_{n+1}/F_n$ for all $n = 1, 2, 3, \ldots$ then the sequence (x_n), where

$$x_1 = a, \quad x_{n+1} = \frac{1}{1 + x_n}, \quad n = 1, 2, \ldots$$

is defined and has a finite limit when n tends to infinity. Determine the limit.

339.1 (Ngô Văn Khương) Given five positive real numbers a, b, c, d, e such that $a^2 + b^2 + c^2 + d^2 + e^2 \leq 1$, prove that

$$\frac{1}{1 + ab} + \frac{1}{1 + bc} + \frac{1}{1 + cd} + \frac{1}{1 + de} + \frac{1}{1 + ea} \geq \frac{25}{6}.$$

339.2 (Lê Chu Biên) Suppose that $ABCD$ is a rectangle. The line perpendicular to AC at C intersects lines AB, AD respectively at E, F. Prove the identity $BE\sqrt{CF} + DF\sqrt{CE} = AC\sqrt{EF}$.

339.3 (Trần Hồng Sơn) Let I be the incenter of triangle ABC and let m_a, m_b, m_c be the lengths of the medians from vertices A, B and C, respectively. Prove that

$$\frac{IA^2}{m_a^2} + \frac{IB^2}{m_b^2} + \frac{IC^2}{m_c^2} \le \frac{3}{4}.$$

339.4 (Quách Văn Giang) Given three positive real numbers a, b, c such that $ab + bc + ca = 1$. Prove that the minimum value of the expression $x^2 + ry^2 + tz^2$ is $2m$, where m is the root of the cubic equation $2x^3 + (r + s + 1)x^2 - rs = 0$ in the interval $(0, \sqrt{rs})$. Find all primes r, s such that $2m$ is rational.

339.5 (Nguyễn Trường Phong) The sequence (x_n) is defined by

$$x_n = a_n^{a_n}, \quad \text{where} \quad a_n = \frac{(2n)!}{(n!)^2 \cdot 2^{2n}}, \quad \text{for } n = 1, 2, 3, \ldots.$$

Prove that the sequence (x_n) has a limit when n tends to infinity and determine the limit.

339.6 (Huỳnh Tấn Châu) Let a be a real number, $a \in (0, 1)$. Determine all functions $f : \mathbb{R} \to \mathbb{R}$ that are continuous at $x = 0$ and satisfy the equation

$$f(x) - 2f(ax) + f(a^2x) = x^2$$

for all real x.

339.7 (Nguyễn Xuân Hùng) In the plane, given a circle with center O and radius r. Let P be a fixed point inside the circle such that $OP = d > 0$. The chords AB and CD through P make a fixed angle α, $(0^\circ < \alpha \le 90^\circ)$. Find the maximum and minimum value of the sum $AB + CD$ when both AB and CD vary, and determine the position of the two chords.

340.1 (Phạm Hoàng Hà) Find the maximum value of the expression

$$\frac{x + y}{1 + z} + \frac{y + z}{1 + x} + \frac{z + x}{1 + y},$$

where x, y, z are real numbers in the interval $[\frac{1}{2}, 1]$.

340.2 (Nguyễn Quỳnh) Let M be a point interior to triangle ABC, let AM intersect BC at E, let CM meet AB at F. Suppose that N is the reflection of B across the midpoint of EF. Prove that the line MN has a fixed point when M moves in the triangle ABC.

340.3 (Trần Tuấn Anh) Let a, b, c be the side lengths of a triangle, and F its area, prove that $F \leq \frac{\sqrt{3}}{4}(abc)^{2/3}$, and determine equality cases.

340.4 (Hàn Ngọc Đức) Given non-negative integers $n, k, n > 1$ and let $\{a_1, a_2, \ldots, a_n\}$ be the n real numbers, prove that

$$\sum_{i=1}^{n} \sum_{j=1}^{n} \frac{a_i a_j}{\binom{k+2}{k+i+j}} \geq 0.$$

340.5 (Trần Minh Hiền) Does there exist a function $f : \mathbb{R}^* \to \mathbb{R}^*$ such that

$$f^2(x) \geq f(x+y)(f(x)+y)$$

for all positive real numbers x, y?

341.1 (Trần Tuyết Thanh) Find all integers x, y, z, t such that

$$x^y + y^z + z^t = x^{2005}.$$

341.2 (Nguyễn Hữu Bằng) Solve the equation

$$(x^2 - 12x - 64)(x^2 + 30x + 125) + 8000 = 0.$$

341.3 (Đoàn Quốc Việt) Given an equilateral triangle ABC, let D be the reflection of B across the line AC. A line through point B intersects the lines AD, CD at M and N respectively, E is the point of intersection of AN and CM. Prove that A, C, D, and E are concyclic.

341.4 (Nguyễn Thanh Nhàn) Prove that for every positive integer $n > 2$, there exist n distinct positive integers such that the sum of these numbers is equal to their least common multiple and is equal to $n!$.

341.5 (Nguyễn Vũ Lươngg) Prove that if $x, y, z > 0$ then

a) $$\sqrt{\frac{x}{y+2z}} + \sqrt{\frac{y}{x+2z}} + 2\sqrt{\frac{z}{x+y+z}} > 2,$$

b) $$\sqrt[3]{\frac{x}{y+2z}} + \sqrt[3]{\frac{y}{x+2z}} + 2\sqrt[3]{\frac{z}{x+y+z}} > 2.$$

342.1 (Trần Văn Hinh) Let ABC be an isosceles triangle with $\angle ABC = \angle ACB = 36°$. Point N is chosen on the angle bisector of $\angle ABC$ such that $\angle BCN = 12°$. Compare the length of CN and CA.

342.2 (Cù Huy Toàn) Find integers (x, y) such that

$$5x^2 + 4y^2 + 5 = (x^2 + y^2 + 1)^2.$$

342.3 (Trần Tuấn Anh) Show that if $a \geq 0$, then

$$\sqrt{9+a} \geq \sqrt{a} + \frac{2\sqrt{2}}{\sqrt{1+a}}.$$

When does the equality hold?

342.4 (Nguyễn Minh Hà) Given an isosceles triangle ABC with $AB = AC$ amd $\angle BAC = 80^\circ$. Point M is interior to the triangle such that $\angle MAC = 20^\circ$ and $\angle MCA = 30^\circ$. Calculate $\angle MBC$.

342.5 (Bùi Văn Chi) Let (ω) be a circle. Suppose that three points A, B, and C on the circle are not diametrically symmetric, and $AB \neq BC$. A line passing through A perpendicular to OB intersects CB at N. Let M be the midpoint of AB, and D be the second intersection of BM and the circle. Suppose that OE is the diameter of the circle through points B, D and the center of the circle. Prove that A, C, E are collinear.

342.6 (Nguyễn Trọng Quân) Let r, R be the inradius, circumradius of a triangle ABC, respectively. Prove that

$$\cos A \cos B \cos C \leq \left(\frac{r}{R\sqrt{2}}\right)^2.$$

342.7 (Phạm Ngọc Bội) Let S be a set of 2005 positive numbers $a_1, a_2, \ldots, a_{2005}$. Let T_i be the non-empty subset of S, s_i be the sum of the numbers belonging T_i. Prove that the set of numbers s_i can be partitioned into 2005 non-empty disjoint subsets so that the ratio of two arbitrary numbers in a subset does not exceed 2.

342.8 (Đỗ Thanh Sơn) Suppose that a, b, c, d are positive real numbers such that $(bc - ad)^2 = 3(ac + bd)^2$. Prove that

$$\sqrt{(a-c)^2 + (b-d)^2} \geq \tfrac{1}{2}\left(\sqrt{a^2+b^2} + \sqrt{c^2+d^2}\right).$$

342.9 (Trần Văn Tân) The sequence (x_n) $(n = 1, 2, \ldots,)$ is defined by $x_1 = 1$, and

$$x_{n+1} = \sqrt{x_n(x_n+1)(x_n+2)(x_n+3)+1}, \quad \text{for } n = 1, 2, \ldots$$

Let

$$y_n = \frac{1}{x_1+2} + \frac{1}{x_2+2} + \cdots + \frac{1}{x_n+2}, \quad (n = 1, 2, \ldots)$$

Find $\lim\limits_{n\to\infty} y_n$.

343.1 (Vĩnh Linh) Triangle ABC has $\angle BAC = 55^\circ$, $\angle ABC = 115^\circ$. A point P is chosen on the internal angle bisector of angle ACB such that $\angle PAC = 25^\circ$. With proof, find $\angle BPC$.

343.2 (Phạm Hoàng Hà) Find all natural numbers x, y, z such that $x^3 + y^3 = 2z^3$, and $x + y + z$ is a prime.

343.3 (Cao Xuân Nam) Let $a, b > 1$ be real numbers such that $a + b \leq 4$, find the minimum value of the expression

$$F = \frac{a^4}{(b-1)^3} + \frac{b^4}{(a-1)^3}.$$

343.4 (Phạm Huy Thông) An isosceles triangle ABC has $BC = a$, $AB = AC = b$, $a > b$. The bisector BD is equal to b, prove that

$$\left(1 + \frac{a}{b}\right)\left(\frac{a}{b} - \frac{b}{a}\right) = 1.$$

343.5 (Nguyễn Việt Hà) A triangle ABC has internal angle bisectors AP, BQ, and CR. Suppose that $\angle PQR = 90^\circ$, find $\angle ABC$, $\angle BCA$, and $\angle CAB$.

343.6 (Phan Thành Nam) For each each positive number x, denote by $a(x)$ the number of prime numbers not exceeding x. For each positive integer m, denote by $b(m)$ the number of prime divisors of m. Prove that for each positive integer n, we have

$$a(n) + a\left(\frac{n}{2}\right) + \cdots + a\left(\frac{n}{n}\right) = b(1) + b(2) + \cdots + b(n).$$

343.7 (Lê Thừa Thành) Without the aid of calculators, find the measure of acute angle x if

$$\cos x = \frac{1}{\sqrt{1 + (2 + \sqrt{3} - \sqrt{2} - \sqrt{6})^2}}.$$

343.8 (Lưu Xuân Tình) Suppose that triangle ABC has $a^2 = 4F \cot A$, where $BC = a$, and F denotes the area of triangle ABC. Let O and G be respectively the circumradius and centroid of triangle ABC. Find the angle between AG and OG.

343.9 (Phan Tuấn Cộng) For a triangle ABC, find A, B, and C such that $\sin^2 A + \sin^2 B - \sin^2 C$ is a minimum.

343.10 (Nguyễn Minh Hà) On the side of triangle ABC, equilateral triangles ABE, ACF are outwardly constructed. Let G be the center of triangle ABC, and K the midpoint of EF. Prove that $\triangle KGC$ is right and one of its angle is 60°.

343.11 (Nguyễn Minh Hà) Let ABC be a triangle with internal angle bisectors AP, BQ, and CR. Let M be any point in the plane of the triangle ABC but not on its sides. Let X, Y, and Z be reflections of M across AP, BQ, and CR. Prove that AX, BY, CZ are either concurent or pairwisely parallel.

343.12 (Nguyễn Minh Hà) Let M be any point in the plane of triangle ABC. Let H, K, L be the projections of M on the lines BC, CA, AB. Find the locus of M such that H, K, L are collinear.

344.1 (Vũ Hữu Chín) Let ABC be a right isosceles triangle with hypothenuse BC. Let M be the midpoint of BC, G be a point chosen on the side AB such that $AG = \frac{1}{3}AB$, E be the foot of the perpendicular from M on CG. Let MG intersect AC at D, compare DE and BC.

344.2 (Hoàng Anh Tuấn) Solve the equation

$$\frac{2+x}{\sqrt{2}+\sqrt{2+x}} + \frac{2-x}{\sqrt{2}-\sqrt{2-x}} = \sqrt{2}.$$

344.3 (Vũ Đức) Sovle the system of equations

$$x^2 + y^2 = 1,$$
$$3x^3 - y^3 = \frac{1}{x+y}.$$

344.4 (Tạ Hoàng Thông) Let a, b, c be positive real numbers such that $a^2 + b^2 + c^2 = 3$, find the greatest possible constant λ such that

$$ab^2 + bc^2 + ca^2 \geq \lambda(ab + bc + ca)^2.$$

344.5 (Hàn Ngọc Đức) Let X be any point on the side AB of the parallelogram $ABCD$. A line through X parallel to AD intersects AC at M nad intersects BD at N; XD meets AC at P and XC cuts BD at Q. Prove that

$$\frac{MP}{AC} + \frac{NQ}{BD} \geq \frac{1}{3}.$$

When does equality hold?

344.6 (Hồ Quang Vinh) Given a triangle ABC with altitudes AM, BN and inscribed circle (Γ), let D be a point on the circle such that D is distinct from A, B and DA and BN have a common point Q. The line DB intersects AM at P. Prove that the midpoint of PQ lies on a fixed line as D varies on the circle (Γ).

344.7 (Lưu Bá Thắng) Let p be an odd prime number, prove that

$$\sum_{j=0}^{p} \binom{j}{p}\binom{j}{p+j} - (2^p + 1)$$

is divisble by p^2.

344.8 (Trần Nguyên An) Let $\{f(x)\}$, $(n = 0, 10, 2, ...)$ be a sequence of functions defined on $[0, 1]$ such that

$$f_0(x) = 0, \quad \text{and} \quad f_{n+1}(x) = f_n(x) + \frac{1}{2}(x - (f(_n(x))^2) \quad \text{for } n = 0, 1, 2, ...$$

Prove that $\dfrac{nx}{2 + n\sqrt{x}} \leq f_n(x) \leq \sqrt{x}$, for $n \in \mathbb{N}, x \in [0, 1]$.

344.9 (Trần Nguyên Bình) Given a polynomial $p(x) = x^2 - 1$, find the number of distinct zeros of the equation

$$p(p(\cdots(p(x))\cdots)) = 0,$$

where there exist 2006 notations of p inside the equation.

344.10 (Nguyễn Minh Hà) Let $ABCDEF$ be a convex inscribable hexagon. The diagonal BF meets AE, AC respectively at M, N; diagonal BD intersects CA, CE at P, Q in that order, diagonal DF cuts EC, EA at R, S respectively. Prove that MQ, NR, and PS are concurrent.

344.11 (Vietnam 1991) Let A, B, C be angles of a triangle, find the minimum of

$$(1 + \cos^2 A)(1 + \cos^2 B)(1 + \cos^2 C).$$

344.12 (Vietnam 1991) Let $x_1, x_2, \ldots, x_n$ be real numbers in the interval $[-1; 1]$, and $x_1 + x_2 + \cdots + x_n = n - 3$, prove that

$$x_1^2 + x_2^2 + \cdots + x_{n-1}^2 + x_n^2 \leq n - 1.$$

345.1 (Trần Tuấn Anh) Let x, y be real numbers in the interval $[0, \frac{1}{\sqrt{2}}]$, find the maximum of

$$p = \frac{x}{1 + y^2} + \frac{y}{1 + x^2}.$$

345.2 (Cù Huy Toàn) Prove that

$$\frac{3\sqrt{3}}{4} \leq \frac{yz}{x(1 + yz)} + \frac{zx}{y(1 + zx)} + \frac{xy}{z(1 + xy)} \leq \frac{1}{4}(x + y + z),$$

where x, y, z are positive real numbers such that $x + y + z = xyz$.

345.3 (Hoàng Hải Dương) Points E, and D are chosen on the sides AB, AC of triangle ABC such that $AE/EB = CD/DA$. Let M be the intersection of BD and CE. Locate E and D such that the area of triangle BMC is a maximum, and determine the area in terms of triangle ABC.

345.4 (Hoàng Trọng Hảo) Find all x such that the following is an integer.

$$\frac{\sqrt{x}}{x\sqrt{x} - 3\sqrt{x} + 3}.$$

345.5 (Lê Hoài Bắc) Let ABC be a triangle inscribable in circle (Γ). Let the bisector of $\angle BAC$ meet the circle at A and D, the circle with center D, diameter D meets the line AB at B and Q, intersects the line AC at C and O. Prove that AO is perpendicular to PQ.

345.6 (Nguyễn Trọng Tuấn) Determine all the non-empty subsets A, B, C of $\mathbb{N}$ such that

i) $A \cap B = B \cap C = C \cap A = \emptyset$;

ii) $A \cup B \cup C = \mathbb{N}$;

iii) For all $a \in A$, $b \in B$, $c \in C$ then $a + c \in A$, $b + c \in B$, and $a + b \in C$.

345.7 (Nguyễn Trọng Hiệp) Find all the functions $f : \mathbb{Z} \to \mathbb{Z}$ satisfying the following conditions

i) $f(f(m-n) = f(m^2) + f(n) - 2n.f(m)$ for all $m, n \in \mathbb{Z}$;

ii) $f(1) > 0$.

345.8 (Nguyễn Đễ) Let AM, BN, CP be the medians of triangle ABC. Prove that if the radius of the incircles of triangles BCN, CAP, and ABM are equal in length, then ABC is an equilateral triangle.

346.1 (Đỗ Bá Chủ) Determine, with proof, the minimum of

$$(x^2+1)\sqrt{x^2+1} - x\sqrt{x^4+2x^2+5} + (x-1)^2.$$

346.2 (Hoàng Hùng) The quadrilateral $ABCD$ is inscribed in the circle (O) and AB intersects CD at some point, let I be the point of intersection of the two diagonals. Let M and N be the midpoints of BC and CD. Prove that if NI is perpendicular to AB then MI is perpendicular to AD.

346.3 (Trần Quốc Hoàn) Given six positive integers a, b, c, d, e, and f such that $abc = def$, prove that

$$a(b^2 + c^2 + d(e^2 + f^2)$$

is a whole number.

346.4 (Bùi Đình Thân) Given quadratic trinomials of the form $f(x) = ax^2 + bx + c$, where a, b, c are integers and $a > 0$, has two distinct roots in the interval $(0, 1)$. Find all the quadratic trinomials and determine the one with the smallest possible leading coefficient.

346.5 (Phạm Kim Hùng) Prove that

$$xy + yz + zx \geq 8(x^2 + y^2 + z^2)(x^2y^2 + y^2z^2 + z^2x^2),$$

where x, y, z are non-negative numbers such that $x + y + z = 1$.

346.6 (Lam Son, Thanh Hoa) Let x, y, z be real numbers greater than 2 such that $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = 1$, prove that

$$(x-2)(y-2)(z-2) \leq 1.$$

346.7 (Huỳnh Duy Thuỷ) Given a polynomial $f(x) = mx^2 + (n-p)x + m + n + p$ with m, n, p being real numbers such that $(m+n)(m+n+p) \leq 0$, prove that

$$\frac{n^2 + p^2}{2} \geq 2m(m+n+p) + np.$$

346.8 (Vũ Thái Lộc) The incircle (I) of a triangle $A_1A_2A_3$ with radius r touches the sides A_2A_3, A_3A_1, A_1A_2 respectively at M_1, M_2, M_3. Let (I_i) be the cirlce touching the sides A_iA_j, A_iA_k and externally touching (I) $(i, j, k \in \{1, 2, 3\},\ i \neq j \neq k \neq i)$. Let K_1, K_2, K_3 be the points of tangency of (I_1) with A_1A_2, of (I_2) with A_2A_3, of (I_3) with A_3A_1 respectively. Let $A_iA_i = a_i,\ A_iK_i = b_i$, $(i = 1, 2, 3)$, prove that

$$\frac{1}{r}\sum_{i=1}^{3}(a_i + b_i) \geq 2 + \sqrt{3}.$$

When does equality hold?

347.1 (Nguyễn Minh Hà) Given a triangle ABC, points E and F are chosen respectively on sides AC and AB such that $\angle ABE = \frac{1}{3}\angle ABC$, $\angle ACF = \frac{1}{3}\angle ACB$. Let O be the intersection of BE and CF. Suppose that $OE = OF$, prove that either $AB = AC$ or $\angle BAC = 90°$.

347.2 Find integer solutions of the system

$$4x^3 + y^2 = 16,$$
$$z^2 + yz = 3.$$

347.3 (Trần Hồng Sơn) The quadratic equation $ax^2 + bx + c = 0$ has two roots in the interval $[0, 2]$. Find the maximum of

$$f = \frac{8a^2 - 6ab + b^2}{4a^2 - 2ab + ac}.$$

347.4 (Nguyễn Lái) $ABCD$ is a quadrilateral, points M, P are chosen on AB and AC such that $AM/AB = CP/CD$. Find all locus of midpoints I of MP as M, P vary on AB, AC.

347.5 (Huỳnh Thanh Tâm) Let ABC be a triangle with $\angle BAC = 135^\circ$, altitudes AM and BN. Line MN intersects the perpendicular bisector of AC at P, let D and E be the midpoints of NP and BC respectively. Prove that ADE is a right isosceles triangle.

347.6 (Nguyễn Sơn Hà) Given 167 sets $A_1, A_2, \ldots, A_{167}$ such that

i) $\sum_{i=1}^{167} |A_i| = 2004$;

ii) $|A_j| = |A_i||A_i \cap A_j|$ for $i, j \in \{1, 2, \ldots, 167\}$ and $i \neq j$,

determine $|\bigcup_{i=1}^{167} A_i|$, where $|A|$ denotes the number of elements of set A.

347.7 (Nguyễn Văn Ái) Find all functions f continuous on $\mathbb{R}$ such that $f(f(f(x))) + f(x) = 2x$, for all x in $\mathbb{R}$.

347.8 (Thái Viết Bảo) Let ABC be an acute-angled triangle with altitudes AD, BE, CF and O is the circumcenter. Let M, N, P be the midpoints of BC, CA, AB. Let D' be the inflection of D across M, E' be the inflection of E across N, F' be the inflection of F across P. Prove that O is interior to triangle $D'E'F'$.

348.1 (Phạm Huy Thông) Find all four-digit numbers $\overline{abcd}$ such that

$$\overline{abcd} = a^2 + 2b^2 + 3c^2 + 4d^2 + 2006.$$

348.2 (Tạ Hoàng Thông) Find the greatest value of the expression

$$p = 3(xy + yz + zx) - xyz,$$

where x, y, z are positive real numbers such that

$$x^3 + y^3 + z^3 = 3.$$

348.3 (Đào Quốc Dũng) ABC is a triangle, let P be a point on the line BC. Point D is chosen on the opposite ray of AP such that $AD = \frac{1}{2}BC$. Let E, F be the midpoints of DB and DC respectively. Prove that the circle with diameter EF has a fixed point when P varies on the line BC.

348.4 (Trần Xuân Uy) Triangle ABC with $AB = AC = a$, and altitude AH. Construct a circle with center A, radius R, $R < a$. From points B and C, draw the tangents BM and CN to this circle (M and N are the points of tangency) so that they are not symmetric with respect to the altitude AH of triangle ABC. Let I be the point of intersection of BM and CN.

1. Find the locus of I when R varies;
2. Prove that $IB.IC = |a^2 - d^2|$ where $AI = d$.

348.5 (Trương Ngọc Bắc) Given n positive real numbers $a_1, a_2, ..., a_n$ such that

$$\sum_{i=1}^{k} a_i \leq \sum_{i=1}^{k} i(i+1), \quad \text{for } k = 1, 2, 3, \cdots, n,$$

prove that

$$\frac{1}{a_1} + \frac{1}{a_2} + \cdots + \frac{1}{a_n} \geq \frac{n}{n+1}.$$

349.1 (Thái Viết Thảo) Prove that in every triangle ABC with sides a, b, c and area F, the following inequalities hold

$$\text{a) } (ab + bc + ca)\sqrt{\frac{abc}{a^3 + b^3 + c^3}} \geq 4F,$$

$$\text{b) } 8R(R - 2r) \geq (a - b)^2 + (b - c)^2 + (c - a)^2.$$

349.2 (Nguyễn Hữu Bằng) Prove that for each positive integer r less than 59, there is a unique positive integer n less than 59 such that $2^n - r$ is divisible by 59.

349.3 (Phạm Văn Thuận) Let a, b, c, d be real numbers such that

$$a^2 + b^2 + c^2 + d^2 = 1,$$

prove that

$$\frac{1}{1 - ab} + \frac{1}{1 - bc} + \frac{1}{1 - cd} + \frac{1}{1 - ca} + \frac{1}{1 - bd} + \frac{1}{1 - da} \leq 8.$$

350.1 (Nguyễn Tiến Lâm) Consider the sum of n terms

$$S_n = 1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \cdots + \frac{1}{1+2+\cdots+n},$$

for $n \in \mathbb{N}$. Find the least rational number r such that $S_n < r$, for all $n \in \mathbb{N}$.

350.2 (Phạm Hoàng Hà) Find the greatest and the least values of

$$\sqrt{2x+1}+\sqrt{3y+1}+\sqrt{4z+1},$$

where x, y, z are nonegative real numbers such that $x+y+z=4$.

350.3 (Mai Quang Thành) Let M be a point interior to the acute-angled triangle ABC such that $\angle MBA = \angle MCA$. Let K, L be the feet of perpendiculars from M to AB, AC respectively. Prove that K, L are equi-distant from the midpoint of BC and the median from M of the triangle MKL has a fixed point when M varies in the interior of triangle ABC.

350.4 (Phạm Tuấn Khải) Let ABC be a right-angled triangle at A, with the altitude AH. A circle passing through B and C intersects AB and AC at M and N respectively. Construct a rectangle $AMDC$. Prove that HN is perpendicular to HD.

350.5 (Nguyễn Trọng Tuấn) Let a be a natural number greater than 1. Consider a nonempty set $A \subset \mathbb{N}$ such that if $k \in A$ then $k+2a \in A$ and $\left[\frac{k}{a}\right] \in A$, where $[x]$ denotes the integer part of x. Prove that $A = \mathbb{N}$.

350.6 (Nguyễn Tài Chung) Find all continuous functions $f : \mathbb{R} \to \mathbb{R}$ such that

$$9f(8x) - 9f(4x) + 2f(2x) = 100x, \quad \forall x \in \mathbb{R}.$$

350.7 (Trần Tuấn Anh) Find the greatest and least values of

$$f = a(b-c)^3 + b(c-a)^3 + c(a-b)^3,$$

where a, b, c are nonegative real numbers such that $a+b+c=1$.

350.8 (Trần Minh Hiền) Let I and G be the incenter and centroid of triangle ABC. Let r_A, r_B, r_C be the circumradius of triangles IBC, ICA, and IAB, respectively; let R_A, R_B, R_C be the circumradius of triangles GBC, GCA, and GAB. Prove that

$$r_A + r_B + r_C \geq R_A + R_B + R_C.$$

351.1 (Mạc Đăng Nghị) Prove that for all real numbers x, y, z

$$(x+y+z)^8 + (y+z-x)^8 + (z+x-y)^8 + (x+y-z)^8 \leq 2188(x^8+y^8+z^8).$$

351.2 (Trần Văn Thính) Find the prime p such that $2005^{2005} - p^{2006}$ is divisible by $2005 + p$.

351.3 (Huỳnh Quang Lâu) Calculate

$$\frac{3^3+1^3}{2^3-1^3}+\frac{5^3+2^3}{3^3-2^3}+\frac{7^3+3^3}{4^3-3^3}+\cdots+\frac{4012^3+2006^3}{2007^3-2006^3}.$$

351.4 (Nguyễn Quang Hưng) Solve the system

$$\begin{aligned} x+y+z+t &= 12,\\ x^2+y^2+z^2+t^2 &= 50,\\ x^3+y^3+z^3+t^3 &= 252,\\ x^2t^2+y^2z^2 &= 2xyzt. \end{aligned}$$

351.5 (Trần Việt Hùng) Five points A, B, C, D, and E are on a circle. Let M, N, P, and Q be the orthogonal projections of E on the lines AB, BC, CD and D. Prove that the orthogonal projections of point E on the lines MN, NP, PQ and QM are concyclic.

351.6 (Phạm Văn Thuận) Prove that if $a, b, c, d \geq 0$ such that

$$a+b+c+d=1,$$

then

$$(a^2+b^2+c^2)(b^2+c^2+d^2)(c^2+d^2+a^2)(d^2+a^2+b^2) \leq \frac{1}{64}.$$

351.7 (Trần Việt Anh) Prove that

$$(2n+1)^{n+1} \leq (2n+1)!!\pi^n$$

for all $n \in \mathbb{N}$, where $(2n+1)!!$ denotes the product of odd positve integers from 1 to $2n+1$.

352.1 (Đỗ Văn Ta) Let a, b, c be positive real numbers such that $abc \geq 1$, prove that

$$\frac{a}{\sqrt{b+\sqrt{ac}}}+\frac{b}{\sqrt{c+\sqrt{ab}}}+\frac{c}{\sqrt{a+\sqrt{bc}}} \geq \frac{3}{\sqrt{2}}.$$

352.2 (Vũ Anh Nam) Let $ABCD$ be a convex function, let E and F be the midpoints of AD, BC respectively. Denote by M the intersection of AF and BE, N the intersection of CE and DF. Find the minimum of

$$\frac{MA}{MF}+\frac{MB}{ME}+\frac{NC}{NE}+\frac{ND}{NF}.$$

352.3 (Hoàng Tiến Trung) Points A, B, C are chosen on the circle O with radius R such that $CB - CA = R$ and $CA.CB = R^2$. Calculate the angle measure of the triangle ABC.

352.4 (Nguyễn Quốc Khánh) Let $\mathbb{N}_m$ be the set of all integers not less than a given integer m. Find all functions $f : \mathbb{N}_m \to \mathbb{N}_m$ such that

$$f(x^2 + f(y)) = y + (f(x))^2, \quad \forall x, y \in \mathbb{N}_m.$$

352.5 (Lê Văn Quang) Suppose that r, s are the only positve roots of the system

$$x^2 + xy + x = 1,$$
$$y^2 + xy + x + y = 1.$$

Prove that

$$\frac{1}{r} + \frac{1}{s} = 8\cos^3\frac{\pi}{7}.$$

352.6 (Trần Minh Hiền) In triangle ABC with $AB = c$, $BC = a$, $CA = b$, let h_a, h_b, and h_c be the altitudes from vertice A, B, and C respectively. Let s be the semiperimeter of triangle ABC. Point X is chosen on side BC such that the inradii of triangles ABX, and ACX are equal, and denote this radius r_A; r_B, and r_C are defined similarly. Prove that

$$2(r_A + r_B + r_C) + s \leq h_a + h_b + h_c.$$

353.1 (Phan Thị Mùi) Do there exist three numbers a, b, c such that

$$\frac{a}{b^2 - ca} - \frac{b}{c^2 - ab} = \frac{c}{a^2 - bc}?$$

353.2 (Nguyễn Tiến Lâm) Find all positive integers x, y, z satisfying simultaneously two conditions.

i) $\dfrac{x - y\sqrt{2006}}{y - z\sqrt{2006}}$ is a rational number.

ii) $x^2 + y^2 + z^2$ is a prime.

353.3 (Vũ Hữu Chín) Let $AA'C'C$ be a convex quadrilateral with I being the intersection of the two diagonals AC and $A'C'$. Point B is chosen on AC and B' chosen on $A'C'$. Let O be the intersection of AC' and $A'C$; P the intersection of AB' and $A'B$; Q the intersection of BC' and $B'C$. Prove that P, O, Q are collinear.

353.4 (Nguyễn Tấn Ngọc) Let ABC be an isosceles triangle with $AB = AC$. Point D is chosen on side AB, E chosen on AC such that $DE = BD + CE$. The bisector of angle $\angle BDE$ meets BC at I.

i) Find the measure of $\angle DIE$.

ii) Prove that DI has a fixed point when D and E vary on AB, and AC, respectively.

353.5 (Trần Quốc Hoàn) Find all positive integers n exceeding 1 such that if $1 < k < n$ and $(k, n) = 1$ for all k, then k is a prime.

353.6 (Phạm Xuân Thịnh) Find all polynomials $p(x)$ such that

$$p(x^{2006} + y^{2006}) = (p(x))^{2006} + (p(y))^{2006},$$

for all real numbers x, y.

353.7

353.8

354.1 (Trần Quốc Hoàn) Find all natural numbers that can be written as the sum of two relatively prime integers greater than 1.

Find all natural numbers, each of which can be written as the sum of three pairwise relatively prime integers greater than 1.

354.2 (Trần Anh Tuấn) Let ABC be a triangle with $\angle ABC$ being acute. Suppose that K be a point on the side AB, and H be its orthogonal projection on the line BC. A ray Bx cuts the segment KH at E and meets the line passing through K parallel to BC at F. Prove that $\angle ABC = 3\angle CBF$ if and only if $EF = 2BK$.

354.3 (Nguyễn Xuân Thủy) Find all natural numbers n such that the product of the digits of n is equal to $(n-86)^2(n^2-85n+40)$.

354.4 (Đặng Thanh Hải) Prove that

$$ab + bc + ca < \sqrt{3}d^2,$$

where a, b, c, d are real numbers such that $0 < a, b, c < d$ and

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{d} - \sqrt{\frac{1}{a^2} + \frac{1}{b^2} + \frac{1}{c^2}} = \frac{2}{d}.$$

354.5 (Lương Văn Bá) Let $ABCD$ be a square with side a. A point M is chosen on the side AD such that $AM = 3MD$. Ray Bx intersects CD at I such that $\angle ABM = \angle MBI$. Suppose that BN is the bisector of angle $\angle CBI$. Calculate the area of triangle BMN.

354.6 (Phạm Thị Bé) Let BC be a fixed chord (distinct from the diameter) of a circle. A point A is chosen on the major arc BC, distinct from the endpoints B, C. Let H be the orthocenter of the triangle ABC. The line BC intersects the circumcircle of triangle ABH and the circumcircle of ACH again at E and F respectively. Let EH meet AC at M, FH intersects AB at N. Locate A such that the measure of the segment MN is a minimum.

354.7 (Đỗ Thanh Hân) Determine the number of all possible natural 9-digit numbers that each has three distinct odd digits, three distinct even digits and every even digit in each number appears exactly two times in this number.

354.8 (Trần Tuấn Anh) For every positive integer n, consider function f_n defined on $\mathbb{R}$ by

$$f_n(x) = x^{2n} + x^{2n-1} + \cdots + x^2 + x + 1.$$

i) Prove that the function f_n has a minimum at only one point.

ii) Suppose that S_n is the minimum at point x_n. Prove that $S_n > \frac{1}{2}$ for all n and there is not a real number $a > \frac{1}{2}$ such that $S_n > a$ for all n. Also prove that (S_n) $(n = 1, 2, ..., n)$ is a decreasing sequence and $\lim S_n = \frac{1}{2}$, and $\lim x_n = -1$.

354.9 (Đàm Huy Đông) Given $x = 20062007$, and let

$$A = \sqrt{x^2 + \sqrt{4x^2 + \sqrt{16x^2 + \sqrt{100x^2 + 39x + \sqrt{3}}}}},$$

find the greatest integer not exceeding A.

354.10 (Tôn Thất Hiệp) i) Find the greatest a such that $3^m \geq m^3 + a$ for all $m \in \mathbb{N}$ and $m \geq 4$.

ii) Find all a such that $n^{n+1} \geq (n+1)^n + a$, for all $n \in \mathbb{N}$, $n \geq 3$.

355.1 (Nguyễn Minh Hà) Let ABC be a right angled triangle with hypothenuse BC and $\angle ABC = 60°$. Point M is chosen on side BC such that $AB + BM = AC + CM$. Find the measure of $\angle CAM$.

355.2 (Dương Châu Dinh) Find all positive integers x, y greater than 1 such that $2xy - 1$ is divisible by $(x-1)(y-1)$.

355.3 (Phan Lê Nhật Duy) Circle (I, r) is externally tangent to circle (J, R) in the point P, and $r \neq R$. Let line IA touch the circle (J, R) at A; JB touch the circle (I, r) at B such that points A, B all belong to the same side of IJ. Points H, K are chosen on IA and JB respectively such that BH, AK are all perpendicular to IJ. Line TH cuts the circle (I, r) again at E, and TK meets the circle (J, R) again at F. Let S be the intersection of EF and AB. Prove that IA, JB, and TS are concurrent.

355.4 (Nguyễn Trọng Tuấn) Let S be a set of 43 positive integers not exceeding 100. For each subset X of S, denote by t_X the product of elements of X. Prove that there exist two disjoint subsets A, B of S such that $t_A t_B^2$ is the cube of a natural number.

355.5 (Phạm Văn Thuận) Find the maximum of the expression

$$\frac{a}{c} + \frac{b}{d} + \frac{c}{a} + \frac{d}{b} - \frac{abcd}{(ab+cd)^2},$$

where a, b, c, d are distinct real numbers such that $ac = bd$, and

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{d} + \frac{d}{a} = 4.$$

355.6 (Phạm Bắc Phú) Let $f(x)$ be a polynomial of degree n with leading coefficient a. Suppose that $f(x)$ has n distinct roots $x_1, x_2, ..., x_n$ all not equal to zero. Prove that

$$\frac{(-1)^{n-1}}{ax_1x_2...x_n} \sum_{k=1}^{n} \frac{1}{x_k} = \sum_{k=1}^{n} \frac{1}{x_k^2 f'(x_k)}.$$

Does there exist a polynomial $f(x)$ of degree n, with leading coefficient $a = 1$, such that $f(x)$ has n distinct roots $x_1, x_2, ..., x_n$, all not equal to zero, satisfying the condition

$$\frac{1}{x_1 f'(x_1)} + \frac{1}{x_2 f'(x_2)} + \cdots + \frac{1}{x_n f'(x_n)} + \frac{1}{x_1x_2...x_n} = 0?$$

355.7 (Ngô Việt Nga) Find the least natural number indivisible by 11 and has the following property: replacing its arbitrary digit by different digit so that the absolute value of their difference is 1 and the resulting number is divisible by 11.

355.8 (Emil Kolev) Consider an acute, scalene triangle ABC. Let H, I, O be respectively its orthocenter, incenter and circumcenter. Prove that there is no vertex or there are exactly two vertices of triangle ABC lying on the circle passing through H, I, O.

355.9 (Trần Nam Dũng) Prove that if $x, y, z > 0$ then

$$xyz + 2(4 + x^2 + y^2 + z^2) \geq 5(x + y + z).$$

When does equality hold?

355.10 (Nguyễn Lâm Chi) Consider a board of size 5×5. Is it possible to color 16 small squares of this board so that in each square of size 2×2 there are at most two small squares which are colored?

355.11 (Nguyễn Khắc Huy) In the plane, there are some points colored red and some colored blue; points with distinct colors are joint so that

i) each red point is joined with one or two read points;

ii) each blue point is joint with one or two red points.

Prove that it is possible to erase less than a half of the given points so that for the remaining points, each blue point is joint with exactly one red point.

356.1 (Lương Văn Bá) Let BE, CF be the altitudes of a triangle ABC. Prove that $AB = AC$ if and only if $AB + BE = AC + CF$.

356.2 (Trần Quốc Hoàn) Let A be a natural number greater than 9 which is formed by using the digits $1, 3, 7, 9$. Prove that A has at least one prime factor greater than or equal to 11.

356.3 (Nguyễn Đăng Phất) Let ABC be a triangle without angle right angle. Let AA', BB', and CC' be the altitudes, D, E, F be the center of escribed circles in the angles $\angle B'AC'$, $\angle C'A'B$, and $\angle B'A'C$ of triangle $AB'C'$, $BC'A'$, $CA'B'$. The escribed circle of $\angle BAC$ of triangle ABC is tangent to BC, CA, AB at M, N, P respectively. Prove that the circumcenter of triangle DEF is the orthocenter of triangle MNP.

356.4 Ten teams participated in a football competition where each team play against every other team exactly once. When the competition was over, it turned out that for every three teams A, B, C, if A defeated B, and B defeated C then A defeated C. Prove that there were four teams A, B, C, D such that

...to be continued

Toan Tuoi Tho Magazine

Vol II, Problems in Toan Tuoi Tho Magazine

Toan tuoi tho is another mathematical monthly magazine intended to be useful to pupils at between 11 and 15 in Vietnam. It is also a readable magazine with various corners and problems in geometry, algebra, number theory.

Now just try some problems in recent issue. Actually there are more, but I do not have enough time.

1.1 (Nguyen Van Manh) Let M be an arbitrary point in triangle ABC. Through point M construct lines DE, IJ, FG such that they are respectively parallel to BC, CA, AB, where $G, J \in BC; E, F \in CA; D, I \in AB$. Prove that

$$(AIMF) + (BGMD) + (CEMJ) \leq \frac{2}{3}(ABC).$$

1.2 (Phan Tien Thanh) Let x, y, z be real number in the interval $(0, 1)$ such that $xyz = (1-x)(1-y)(1-z)$. Prove that

$$x^2 + y^2 + z^2 \geq \frac{3}{4}.$$

1.3 (Nguyen Trong Tuan) Given a natural three digit number, we can change the given number in two following possible ways:

i) take the first digit (or the last digit) and insert it into other two;

ii) reverse the order of the digits.

After 2005 times of so changing, can we obtain the number 312 from the given number 123?

1.4 (Nguyen Minh Ha) Three circles (O_1), (O_2), (O_3) intersect in one point O. Three points A_1, A_2, A_3 line on the circles $(O_1), (O_2), (O_3)$ respectively such that OA_1, OA_2, OA_3 are parallel to O_2O_3, O_3O_1, O_1O_2 in that order. Prove that O, A_1, A_2, A_3 are concyclic.

1.5 (Nguyen Ba Thuan) Let ABC be a scalene triangle $AB \neq AC$ inscribed in triangle (O). The circle (O') is internally tangent to (O) at T, and AB, AC at E, F respectively. AO' intersects (O) at M, distinct from A. Prove that BC, EF, MT are concurrent.

1.6 (Tran Xuan Dang) Solve simultaneous equations

$$x^3 + 2x^2 + x - 3 = y,$$
$$y^3 + 2y^2 + y - 3 = z,$$
$$z^3 + 2z^2 + z - 3 = x.$$

1.7 (Nguyen Huu Bang) Let a, b be nonegative real numbers and

$$p(x) = (a^2 + b^2)x^2 - 2(a^3 + b^3)x + (a^2 - b^2)^2.$$

Prove that $p(x) \leq 0$ for all x satisfying $|a - b| \leq x \leq a + b$.

1.8 (Le Viet An) Let $ABCD$ be a convex quadrilateral, I, J be the midpoints of diagonals AC and BD respectively. Denote $E = AJ \cap BI$, $F = CJ \cap DI$, let H, K be the midpoints of AB, CD. Prove that $EF \parallel HK$.

360.1 (Trần Văn Hinh) ABC is a right angled triangle with hypotenuse BC, $\angle ACB = 54°$. Point E is chosen on the opposite ray of CA such that $\angle ABE = 54°$. Prove that $BC < AE$.

360.2 (Phan Thị Mùi) Let a, b, c be real numbers greater than or equal to $-\frac{3}{2}$ such that

$$abc + ab + bc + ca + a + b + c \geq 0.$$

Prove that

$$a + b + c \geq 0.$$

360.3 (Phạm Huy Thông) Points M, N, P are on the exterior of triangle ABC such that $\angle CAN = \angle CBM = 30°$, $\angle ACN = \angle BCM = 20°$, $\angle PAB = \angle PBA = 40°$. Prove that triangle MNP is equilateral.

360.4 (Hoàng Ngọc Cảnh) Let ABC be a right angled triangle with hypotenuse BC and altitude AH. Let I, I_1, I_2 be the centers of circles inscribed in triangles ABC, AHB, AHC. Prove that the circumcircle of triangle II_1I_2 is congruent to incircle of triangle ABC.

360.5 (Huỳnh Tấn Châu) Find all functions $f : \mathbb{R}^* \to \mathbb{R}^*$ such that

$$f(x).f(y) = f(x + yf(x)), \ \ \forall x, y \in \mathbb{R}^*.$$

360.6 (Nguyễn Bá Thành) Given is an increasing sequence (u_k), $k = 1, 2, ..., n$. A set A_n consists of positive numbers $u_i - u_j$ $(1 \leq j < i \leq n)$. Prove that if the number of elements of A_n is less than n then (u_k), $k = 1, 2, ..., n$ is an arithmetic sequence.

. . . to be continued