Topic Of Nhochnhoc

Compiled and translated by: S. Mukherjee (Potla)
Email : sayanmukherjee1995@gmail.com

1, For positive reals a, b, c prove that:

$$(a+b+c)^3 \geq 6\sqrt{3}(a-b)(b-c)(c-a)$$

2, For nonnegative reals a, b, c prove that

$$\frac{ab}{(a+b)^2}+\frac{bc}{(b+c)^2}+\frac{ca}{(c+a)^2} \leq \frac{1}{4}+\frac{4abc}{(a+b)(b+c)(c+a)}$$

3, For $a, b, c > 0$ satisfying $a^2+b^2+c^2=6$ Find P_{min} where

$$P=\frac{a}{bc}+\frac{2b}{ca}+\frac{5c}{ab}$$

4, For nonnegative reals a, b, c Prove that:

$$\frac{(a+b)^2(a+c)^2}{(b^2-c^2)^2}+\frac{(b+c)^2(a+b)^2}{(c^2-a^2)^2}+\frac{(b+c)^2(c+a)^2}{(a^2-b^2)^2} \geq 2$$

5, For positive reals a, b, c prove that

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}+\frac{16}{5}.\frac{ab+bc+ca}{a^2+b^2+c^2} \geq \frac{18}{5}$$

6,For positive a, b, c;show that :

$$\frac{(a^2+bc)(b^2+ca)(c^2+ab)}{(a^2+b^2)(b^2+c^2)(c^2+a^2)}+\frac{(a-b)(a-c)}{b^2+c^2}+\frac{(b-c)(b-a)}{c^2+a^2}+\frac{(c-a)(c-b)}{a^2+b^2} \geq 1$$

7,For positive reals a, b, c prove that :

$$1+\frac{ab+bc+ca}{a^2+b^2+c^2} \geq \frac{16abc}{(a+b)(b+c)(c+a)}$$

8, For nonnegative a, b, c prove that :

$$(a^2+b^2+c^2-1)^2 \geq 2(a^3b+b^3c+c^3a-1)$$

9, Let $a, b, c \geq 0$ satisfy: $a^2+b^2+c^2=2$ Find P_{max} if

$$P=(a^5+b^5)(b^5+c^5)(c^5+a^5)$$

10, $a, b, c \geq 0$. Prove that

$$(a^2+5bc)(b^2+5bc)(c^2+5ab) \geq 27abc(a+b)(b+c)(c+a)$$

11, (Vasile Cirtoaje) $a, b, c \geq 0$. Prove that:

$$(2a^2 + 7bc)(2b^2 + 7ca)(2c^2 + 7ab) \geq 27(ab + bc + ca)^3$$

12, $a, b, c \geq 0; a + b + c = 3$ Prove that

$$a^3 + b^3 + c^3 + 9abc \leq 2[ab(a + b) + bc(b + c) + ca(c + a)]$$

13, Let $a, b, c > 0$. Show that:

$$\frac{a^2}{2a^2 + (b + c - a)^2} + \frac{b^2}{2b^2 + (c + a - b)^2} + \frac{c^2}{2c^2 + (a + b - c)^2} \leq 1$$

14,Let$a, b, c \geq 0$ Show that:

$$\frac{3a^2 + 5ab}{(b + c)^2} + \frac{3b^2 + 5bc}{(c + a)^2} + \frac{3c^2 + 5ca}{(a + b)^2} \geq 6$$

15, For $a, b, c \geq 0$ and $k \in \mathbf{R}$ find the best constant that satisfies

$$(a + b + c)^5 \geq k(a^2 + b^2 + c^2)(a - b)(b - c)(c - a)$$

16, Let $a, b, c > 0$ satisfy $a + b + c = 3$. Prove that:

$$(a^3 + b^3 + c^3)(ab + bc + ca)^8 \leq 3^9$$

17, $a, b, c \geq 0$ Show that

$$\frac{a^2}{(b + c)^2} + \frac{b^2}{(c + a)^2} + \frac{c^2}{(a + b)^2} + \frac{10abc}{(a + b)(b + c)(c + a)} \geq 2$$

18, For $a, b, c \geq 0$ such that $a + b + c = 3$, prove the following inequality:

$$(ab^3 + bc^3 + ca^3)(ab + bc + ca) \leq 16$$

19,$a, b, c > 0$ satisfy $abc = 1$. Prove that

$$\frac{a}{\sqrt{b^2 + 2c}} + \frac{b}{\sqrt{c^2 + 2a}} + \frac{c}{\sqrt{a^2 + 2b}} \geq \sqrt{3}$$

20, For nonnegative a, b, c satisfying $ab + bc + ca = 3$, prove that

$$3(a + b + c) + 2(\sqrt{a} + \sqrt{b} + \sqrt{c}) \geq 15$$

21, For nonnegative reals a, b, c prove that:

$$\frac{a^2}{(b + c)^2} + \frac{b^2}{(c + a)^2} + \frac{c^2}{(a + b)^2} + \frac{1}{2} \geq \frac{5}{4} \frac{a^2 + b^2 + c^2}{ab + bc + ca}$$

22, For nonnegative a, b, c; show that

$$3(a^4 + b^4 + c^4) + 7(a^2b^2 + b^2c^2 + c^2a^2) \geq 2(a^3b + b^3c + c^3a) + 8(ab^3 + bc^3 + ca^3)$$

23, For $a,b,c \ge 0$, show that:

$$\frac{a^4}{(a+b)^4}+\frac{b^4}{(b+c)^4}+\frac{c^4}{(c+a)^4}+\frac{3abc}{2(a+b)(b+c)(c+a)} \ge \frac{3}{8}$$

24, Let $a,b,c \ge 0$ satisfy $a+b+c=3$ Prove that:

$$\sqrt[3]{\frac{a^3+4}{a^2+4}}+\sqrt[3]{\frac{b^3+4}{b^2+4}}+\sqrt[3]{\frac{c^3+4}{c^2+4}} \ge \ 3$$

25, For positive reals a,b,c show that:

$$5+\frac{3abc}{a^3+b^3+c^3} \ge \ 4\left(\frac{ab}{a^2+b^2}+\frac{bc}{b^2+c^2}+\frac{ca}{c^2+a^2}\right)$$

26, For nonnegative a,b,c show that:

$$\frac{a^2+b^2+c^2}{ab+bc+ca}+\frac{a(b+c)}{b^2+c^2}+\frac{b(c+a)}{c^2+a^2}+\frac{c(a+b)}{a^2+b^2} \ge 4$$

27,For nonnegative reals a,b,c prove that:

$$\frac{(a-b)^2}{(a+b)^2}+\frac{(b-c)^2}{(b+c)^2}+\frac{(c-a)^2}{(c+a)^2}+\frac{24(ab+bc+ca)}{(a+b+c)^2} \le \ 8$$

28, Let $a,b,c \ge 0$; show that :

$$1+\frac{abc}{a^3+b^3+c^3} \ge \ \frac{32abc}{3(a+b)(b+c)(c+a)}$$

29, For nonnegative reals a,b,c show that:

$$\sqrt[3]{\frac{(a^2+bc)(b+c)}{a(b^2+c^2)}}+\sqrt[3]{\frac{(b^2+ca)(c+a)}{b(c^2+a^2)}}+\sqrt[3]{\frac{(c^2+ab)(a+b)}{c(a^2+b^2)}} \ge \ 3\sqrt[3]{2}$$

30, For nonnegative a,b,c show that

$$\frac{a}{\sqrt{b^2+bc+c^2}}+\frac{b}{\sqrt{c^2+ca+a^2}}+\frac{c}{\sqrt{a^2+ab+b^2}} \ge \ \frac{a+b+c}{\sqrt{ab+bc+ca}}$$

31, For nonnegative a,b,c show that

$$(a+b+c)^5 \ge \ 25\sqrt{5}(ab+bc+ca)(a-b)(b-c)(c-a)$$

32,For nonnegative a,b,c show that

$$(a^2+b^2+c^2)^3 \ge \ 27(a-b)^2(b-c)^2(c-a)^2$$

33, For nonnegative a,b,c show that

$$(a^2+b^2+c^2)^3 \ge \ 2(a-b)^2(b-c)^2(c-a)^2$$

34,For nonnegative a, b, c show that

$$\sqrt[3]{\frac{a^5(b+c)}{(b^2+c^2)(a^2+bc)^2}}+\sqrt[3]{\frac{b^5(c+a)}{(c^2+a^2)(b^2+ca)^2}}+\sqrt[3]{\frac{c^5(a+b)}{(a^2+b^2)(c^2+ab)^2}} \geq \frac{3}{\sqrt[3]{4}}$$

35, If A, B, C are three angles of an acute triangle, fing P_{min} where:

$$P=\frac{1}{\sin^n A}+\frac{1}{\sin^n B}+\frac{1}{\sin^n C}+\cos^m A\cos^m B\cos^m C$$

36, For nonnegative a, b, c show that

$$(a^2+5b^2)(b^2+5c^2)(c^2+5a^2) \geq 8abc(a+b+c)^3$$

37, For nonnegative reals a, b, c prove that:

$$1+\frac{8abc}{(a+b)(b+c)(c+a)} \geq \frac{2(ab+bc+ca)}{a^2+b^2+c^2}$$

38, For nonnegative reals a, b, c prove that:

$$3+\frac{8abc}{(a+b)(b+c)(c+a)} \geq \frac{12(ab+bc+ca)}{(a+b+c)^2}$$

39,For nonnegative reals a, b, c prove that:

$$\sqrt{(a+b+c)(ab+bc+ca)} \geq \sqrt{abc}+\sqrt{\frac{(a+b)(b+c)(c+a)}{2}}$$

40, $a, b, c \geq 0$ Show that

$$\frac{a^2+b^2+c^2}{ab+bc+ca}+\frac{1}{2} \geq \frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}$$

41,$a, b, c \geq 0$ Show that if $a+b+c=5$; we have:

$$10+ab^2+bc^2+ca^2 \geq \frac{7}{8}(a^2b+b^2c+c^2a)$$

42, $a, b, c \geq 0$ Show that

$$\frac{a^2}{(a-b)^2}+\frac{b^2}{(b-c)^2}+\frac{c^2}{(c-a)^2} \geq 1$$

43, $a, b, c \geq 0$ Show that if they satisfy $a+b+c=3$ we always have:

$$\frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b}+abc \geq 4$$

44, $a, b, c \geq 0$ Show that

$$\frac{5a^2+2bc}{(b+c)^2}+\frac{5b^2+2ca}{(c+a)^2}+\frac{5c^2+2ab}{(a+b)^2} \geq \frac{21}{4}.\frac{a^2+b^2+c^2}{ab+bc+ca}$$

45, $a, b, c \geq 0$ Show that

$$\frac{3a^2+4bc}{(b+c)^2}+\frac{3b^2+4ca}{(c+a)^2}+\frac{3c^2+4ab}{(a+b)^2} \geq \frac{7}{4}.\frac{(a+b+c)^2}{ab+bc+ca}$$

46, $a, b, c \geq 0$; $a+b+c = 2\sqrt[3]{12}$. Show that:

$$\sqrt[7]{1+a^3}(1+b^3)(1+c^3) \leq 169$$

47, For $a, b, c > 0$ satisfying $a+b+c = 3$; show that:

$$\frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b}+\frac{9abc}{4} \geq \frac{21}{4}$$

48,For $a, b, c > 0$ satisfying $a+b+c = 3$, prove that if $k = \frac{10+4\sqrt{6}}{3}$ we have:

$$\sqrt{3(a^2+b^2+c^2)}+abc \geq 1+\sqrt{3k}$$

49, For $a, b, c > 0$ satisfying $a+b+c = 3.$, show that

$$a\sqrt{a+b}+b\sqrt{b+c}+c\sqrt{c+a} \geq 3\sqrt{2}$$

50, For $a, b, c > 0$ satisfying $a+b+c = 6$, Show that

$$(11+a^2)(11+b^2)(11+c^2)+120abc \geq 4320$$

51,For $a, b, c > 0$ satisfying $ab+bc+ca = 2$, show that

$$ab(4a^2+b^2)+bc(4b^2+c^2)+ca(4c^2+a^2)+7abc(a+b+c) \geq 16$$

52, For $a, b, c > 0$, show that :

$$a\sqrt{a^2+3bc}+b\sqrt{b^2+3ca}+c\sqrt{c^2+3ab} \geq 2(ab+bc+ca)$$

53,For $a, b, c > 0$, prove that

$$a\sqrt{4a^2+5bc}+b\sqrt{4b^2+5ca}+c\sqrt{4c^2+5ab} \geq (a+b+c)^2$$

54, For positive real numbers a, b, c show that

$$\frac{a}{\sqrt{4a^2+5bc}}+\frac{b}{\sqrt{4b^2+5ca}}+\frac{c}{\sqrt{4c^2+5ab}} \leq 1$$

55, For positive real numbers a, b, c show that

$$\frac{a}{a+\sqrt{a^2+3bc}}+\frac{b}{b+\sqrt{b^2+3ca}}+\frac{c}{c+\sqrt{c^2+3ab}} \leq 1$$

56, For positive real numbers a, b, c such that $ab+bc+ca = 1$; show that

$$\frac{1}{\sqrt{a^2+b^2}}+\frac{1}{\sqrt{b^2+c^2}}+\frac{1}{\sqrt{c^2+a^2}} \geq 2+\frac{1}{\sqrt{2}}$$

57, For positive real numbers a, b, c satisfying $a+b+c=2$; show that

$$\frac{1}{\sqrt{a^2+b^2}}+\frac{1}{\sqrt{b^2+c^2}}+\frac{1}{\sqrt{c^2+a^2}} \geq 2+\frac{1}{\sqrt{2}}$$

58, For positive real numbers a, b, c such that $ab+bc+ca=3$, show that

$$\frac{a}{b^3+abc}+\frac{b}{c^3+abc}+\frac{c}{a^3+abc} \geq \frac{3}{2}$$

59, For positive real numbers a, b, c that satisfy $a+b+c=3$, show that

$$\frac{a^2}{b^3+abc}+\frac{b^2}{c^3+abc}+\frac{c^2}{a^3+abc} \geq \frac{3}{2}$$

60, For positive real numbers a, b, c that satisfy $a+b+c=3$, show that

$$(1+a^2)(1+b^2)(1+c^2) \geq (1+a)(1+b)(1+c)$$

61, For positive real numbers a, b, c , show that

$$\frac{a^3+b^3+c^3}{abc}+\frac{24abc}{(a+b)(b+c)(c+a)} \geq 4(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b})$$

62, For positive real numbers a, b, c , show that

$$\frac{a(b+c)}{b^2+c^2}+\frac{b(c+a)}{c^2+a^2}+\frac{c(a+b)}{a^2+b^2} \geq \frac{(a+b+c)^2}{a^2+b^2+c^2}$$

63, For positive real numbers a, b, c , show that

$$a+b+c \geq \frac{a(b+c)}{\sqrt{a^2+3bc}}+\frac{b(c+a)}{\sqrt{b^2+3ca}}+\frac{c(a+b)}{\sqrt{c^2+3ab}}$$

64, For positive real numbers a, b, c , show that

$$a^k+b^k+c^k \geq \frac{a(b^k+c^k)}{\sqrt{a^2+3bc}}+\frac{b(c^k+a^k)}{\sqrt{b^2+3ca}}+\frac{c(a^k+b^k)}{\sqrt{c^2+3ab}}$$

65, For positive real numbers a, b, c , show that

$$\sqrt{a^2+4bc}+\sqrt{b^2+4ca}+\sqrt{c^2+4ab} \geq \sqrt{15(ab+bc+ca)}$$

66, For positive real numbers a, b, c , show that

$$\sqrt{\frac{a^3}{b^3+abc}}+\sqrt{\frac{b^3}{c^3+abc}}+\sqrt{\frac{c^3}{a^3+abc}} \geq \frac{3}{\sqrt{2}}$$

67, For positive real numbers a, b, c , show that

$$(a+b)^2(b+c)^2(c+a)^2 \geq \frac{64}{3}abc(a^2b+b^2c+c^2a)$$

68, For positive real numbers a, b, c , show that

$$\frac{3a^3+abc}{b^3+c^3}+\frac{3b^3+abc}{c^3+a^3}+\frac{3c^3+abc}{a^3+b^3}\geq 6$$

69, For positive real numbers a, b, c , show that

$$(a^2+b^2+c^2)(a+b+c)\geq\ 3\sqrt{3abc(a^3+b^3+c^3)}$$

70, For positive real numbers a, b, c , show that

$$\frac{(a+b+c)^2}{ab+bc+ca}\geq\ \frac{a(b+c)}{a^2+bc}+\frac{b(c+a)}{b^2+ca}+\frac{c(a+b)}{c^2+ab}\geq\ \frac{(a+b+c)^2}{a^2+b^2+c^2}$$

71, For positive real numbers a, b, c , show that

$$\frac{(a+b+c)^2}{2(ab+bc+ca)}\geq\frac{a^2}{a^2+bc}+\frac{b^2}{b^2+ca}+\frac{c^2}{c^2+ab}$$

Also show that

$$\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b}\geq\frac{a^2}{a^2+bc}+\frac{b^2}{b^2+ca}+\frac{c^2}{c^2+ab}$$

72, For positive real numbers a, b, c , show that

$$3(a^2b+b^2c+c^2a)(ab^2+bc^2+ca^2)\geq abc(a+b+c)^3$$

73, For positive real numbers a, b, c , show that

$$\frac{a^2}{b^2}+\frac{b^2}{c^2}+\frac{c^2}{a^2}\geq\frac{(a+b+c)^3}{3(ab^2+bc^2+ca^2)}$$

74, For positive real numbers a, b, c , show that

$$\frac{1}{8a^2+bc}+\frac{1}{8b^2+ca}+\frac{1}{8c^2+ab}\geq\frac{1}{ab+bc+ca}$$

75, For positive reals a, b, c , show that

$$\frac{3(a^2+b^2+c^2)}{a+b+c}\geq\sqrt{a^2-ab+b^2}+\sqrt{b^2-bc+c^2}+\sqrt{c^2-ca+a^2}\geq\sqrt{3(a^2+b^2+c^2)}$$

76, For positive real numbers a, b, c , show that

$$\sqrt{a^2-ab+b^2}+\sqrt{b^2-bc+c^2}+\sqrt{c^2-ca+a^2}\leq 3$$

77, For positive real numbers a, b, c satisfying $a+b+c=3$, show that

$$\sqrt{a^2b+b^2c}+\sqrt{b^2c+c^2a}+\sqrt{c^2a+a^2b}\leq 3\sqrt{2}$$

78, For positive real numbers a, b, c satisfying $a+b+c=3$, show that

$$\frac{a}{\sqrt{b+c^2}}+\frac{b}{\sqrt{c+a^2}}+\frac{c}{\sqrt{a+b^2}} \geq \frac{3}{\sqrt{2}}$$

79, For positive reals a, b, c , show that

$$\frac{3(a+b+c)}{2(ab+bc+ca)} \geq \frac{a}{a^2+b^2}+\frac{b}{b^2+c^2}+\frac{c}{c^2+a^2}$$

80, For positive reals a, b, c , show that

$$\frac{ab}{\sqrt{ab+2c^2}}+\frac{bc}{\sqrt{bc+2a^2}}+\frac{ca}{\sqrt{ca+2b^2}} \geq \sqrt{ab+bc+ca}$$

81, For $a, b, c > 0$ satisfying $a+b+c=2\sqrt[3]{12}$, show that

$$\sqrt[7]{1+a^3}(1+b^3)(1+c^3) \leq 169$$

82, For positive reals a, b, c such that $a+b+c=3$, prove that

$$\frac{ab}{c}+\frac{bc}{a}+\frac{ca}{b}+abc \geq 4$$

83, For positive reals a, b, c such that $a+b+c=3$, prove that

$$\sqrt{3(a^2+b^2+c^2)}+abc \geq 1+\sqrt{3k}$$

where

$$k=\frac{10+4\sqrt{6}}{3}$$

84, For positive reals a, b, c such that $a+b+c=3$, prove that

$$a\sqrt{a+b}+b\sqrt{b+c}+c\sqrt{c+a} \geq 3\sqrt{2}$$

85, (For positive reals a, b, c such that $a+b+c=6$, prove that

$$(11+a^2)(11+b^2)(11+c^2)+120abc \geq 4320$$

86, For positive reals a, b, c such that $ab+bc+ca=2$, prove that

$$ab(4a^2+b^2)+bc(4b^2+c^2)+ca(4c^2+a^2)+7abc(a+b+c) \geq 16$$

87, For positive reals a, b, c , prove that

$$a\sqrt{a^2+3bc}+b\sqrt{b^2+3ca}+c\sqrt{c^2+3ab} \geq 2(ab+bc+ca)$$

88, For positive reals a, b, c , prove that

$$a\sqrt{4a^2+5bc}+a\sqrt{a^2+3bc}+a\sqrt{a^2+3bc} \geq (a+b+c)^2$$

89, For positive reals a, b, c , prove that

$$\frac{a}{\sqrt{4a^2+5bc}}+\frac{b}{\sqrt{4b^2+5ca}}+\frac{c}{\sqrt{4c^2+5ab}}\leq\ 1$$

90, For positive reals a, b, c , prove that

$$\frac{a}{a+\sqrt{a^2+3bc}}+\frac{b}{b+\sqrt{b^2+3ca}}+\frac{c}{c+\sqrt{c^2+3ab}}\leq\ 1$$

91, For positive reals a, b, c satisfying $ab+bc+ca=1$, show that

$$\frac{1}{\sqrt{a^2+b^2}}+\frac{1}{\sqrt{b^2+c^2}}+\frac{1}{\sqrt{c^2+a^2}}\geq 2+\frac{1}{\sqrt{2}}$$

92, For positive reals a, b, c satisfying $a+b+c=2$, show that

$$\frac{1}{\sqrt{a^2+b^2}}+\frac{1}{\sqrt{b^2+c^2}}+\frac{1}{\sqrt{c^2+a^2}}\geq 2+\frac{1}{\sqrt{2}}$$

93, For positive reals a, b, c satisfying $a+b+c=3$, show that

$$\frac{a}{b^3+abc}+\frac{b}{c^3+abc}+\frac{c}{a^3+abc}\geq\frac{3}{2}$$

94, For positive reals a, b, c satisfying $a+b+c=3$, show that

$$\frac{a^2}{b^3+abc}+\frac{b^2}{c^3+abc}+\frac{c^2}{a^3+abc}\geq\frac{3}{2}$$

95,For positive reals a, b, c satisfying $a+b+c=3$, show that

$$(1+a^2)(1+b^2)(1+c^2)\geq(1+a)(1+b)(1+c)$$

96, For positive reals a, b, c show that

$$\sum\frac{1}{a^2+bc}\leq\frac{3\sqrt{3}}{2\sqrt{abc(a+b+c)}}$$

97, Given that

$$\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\geq\frac{9(a^3+b^3+c^3)}{(a^2+b^2+c^2)^2},$$

Prove that

$$(a^2+b^2+c^2)^2(ab+bc+ca)\geq 9abc(a^3+b^3+c^3)$$

98, For $a, b, c>0$, show that

$$a\sqrt{a^2+3bc}+b\sqrt{b^2+3ca}+c\sqrt{c^2+3ab}\geq 2(ab+bc+ca)$$

99, For $a, b, c > 0$, show that

$$a\sqrt{4a^2+5bc}+a\sqrt{a^2+3bc}+a\sqrt{a^2+3bc}\geq(a+b+c)^2$$

100, For $a, b, c > 0$, show that

$$\frac{a}{\sqrt{4a^2+5bc}}+\frac{b}{\sqrt{4b^2+5ca}}+\frac{c}{\sqrt{4c^2+5ab}}\leq 1$$

101, For $a, b, c > 0$, show that

$$\frac{a}{a+\sqrt{a^2+3bc}}+\frac{b}{b+\sqrt{b^2+3ca}}+\frac{c}{c+\sqrt{c^2+3ab}}\leq 1$$

102, For $a, b, c > 0$, show that

$$\frac{a^3+b^3+c^3}{abc}+\frac{24abc}{(a+b)(b+c)(c+a)}\geq 4(\frac{a}{b+c}+\frac{b}{c+a}+\frac{c}{a+b})$$

103, For $a, b, c > 0$, show that

$$\frac{a(b+c)}{b^2+c^2}+\frac{b(c+a)}{c^2+a^2}+\frac{c(a+b)}{a^2+b^2}\geq\frac{(a+b+c)^2}{a^2+b^2+c^2}$$

104, For $a, b, c > 0$, show that

$$a+b+c\geq\frac{a(b+c)}{\sqrt{a^2+3bc}}+\frac{b(c+a)}{\sqrt{b^2+3ca}}+\frac{c(a+b)}{\sqrt{c^2+3ab}}$$

105, For $a, b, c > 0$, show that

$$a^k+b^k+c^k\geq\frac{a(b^k+c^k)}{\sqrt{a^2+3bc}}+\frac{b(c^k+a^k)}{\sqrt{b^2+3ca}}+\frac{c(a^k+b^k)}{\sqrt{c^2+3ab}}$$

106, For $a, b, c > 0$, show that

$$\sqrt{a^2+4bc}+\sqrt{b^2+4ca}+\sqrt{c^2+4ab}\geq\sqrt{15(ab+bc+ca)}$$

107, For $a, b, c > 0$, show that

$$\sqrt{\frac{a^3}{b^3+abc}}+\sqrt{\frac{b^3}{c^3+abc}}+\sqrt{\frac{c^3}{a^3+abc}}\geq\frac{3}{\sqrt{2}}$$

108, For $a, b, c > 0$, show that

$$(a+b)^2(b+c)^2(c+a)^2\geq\frac{64}{3}abc(a^2b+b^2c+c^2a)$$

109, For $a, b, c > 0$, show that

$$\frac{3a^3+abc}{b^3+c^3}+\frac{3b^3+abc}{c^3+a^3}+\frac{3c^3+abc}{a^3+b^3}\geq 6$$

110, For $a, b, c > 0$, show that

$$(a^2+b^2+c^2)(a+b+c) \geq 3\sqrt{3abc(a^3+b^3+c^3)}$$

111, For $a, b, c > 0$, show that

$$\frac{(a+b+c)^2}{ab+bc+ca} \geq \frac{a(b+c)}{a^2+bc} + \frac{b(c+a)}{b^2+ca} + \frac{c(a+b)}{c^2+ab} \geq \frac{(a+b+c)^2}{a^2+b^2+c^2}$$

112, For $a, b, c > 0$, show that

$$\frac{(a+b+c)^2}{2(ab+bc+ca)} \geq \frac{a^2}{a^2+bc} + \frac{b^2}{b^2+ca} + \frac{c^2}{c^2+ab}$$

Also prove that:

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{a^2}{a^2+bc} + \frac{b^2}{b^2+ca} + \frac{c^2}{c^2+ab}$$

113, For $a, b, c > 0$, show that

$$3(a^2b+b^2c+c^2a)(ab^2+bc^2+ca^2) \geq abc(a+b+c)^3$$

114, For $a, b, c > 0$, show that

$$\frac{a^2}{b^2} + \frac{b^2}{c^2} + \frac{c^2}{a^2} \geq \frac{(a+b+c)^3}{3(ab^2+bc^2+ca^2)}$$

115, For $a, b, c > 0$ satisfying $a+b+c=3$, show that

$$\frac{a}{\sqrt{b+c^2}} + \frac{b}{\sqrt{c+a^2}} + \frac{c}{\sqrt{a+b^2}} \geq \frac{3}{\sqrt{2}}$$

116, For $a, b, c > 0$, show that

$$\frac{1}{a\sqrt{a+b}} + \frac{1}{b\sqrt{b+c}} + \frac{1}{c\sqrt{c+a}} \geq \frac{3}{\sqrt{2abc}}$$

117, For $a, b, c > 0$, show that

$$\frac{3a^3+abc}{b^3+c^3} + \frac{3b^3+abc}{c^3+a^3} + \frac{3c^3+abc}{a^3+b^3} \geq 6$$

118, For $a, b, c > 0$, show that

$$\frac{ab}{(ab+2c^2)^5} + \frac{bc}{(bc+2a^2)^5} + \frac{ca}{(ca+2b^2)^5} \geq \frac{1}{(ab+bc+ca)^4}$$

119, For positives a, b, c satisfying $a+b+c=3$, prove that

$$\frac{a}{\sqrt{b+c^2}} + \frac{b}{\sqrt{c+a^2}} + \frac{c}{\sqrt{a+b^2}} \geq \frac{3}{\sqrt{2}}$$

Also show that :

$$\frac{a}{b+c^2}+\frac{b}{c+a^2}+\frac{c}{a+b^2}\geq\frac{9}{3+a+b+c}$$

120, For positives a, b, c, prove that

$$\bullet\sqrt{\frac{a}{b+3c}}+\sqrt{\frac{b}{c+3a}}+\sqrt{\frac{c}{a+3b}}\geq\frac{3}{2}$$

$$\bullet\sqrt{\frac{a}{b+2c}}+\sqrt{\frac{b}{c+2a}}+\sqrt{\frac{c}{a+2b}}\geq\sqrt[4]{8}$$

121, For positives a, b, c, prove that

$$\bullet\frac{a}{\sqrt{ab+8c^2}}+\frac{b}{\sqrt{bc+8a^2}}+\frac{c}{\sqrt{ca+8b^2}}\geq 1$$

$$\bullet\frac{a}{\sqrt{ab+c^2}}+\frac{b}{\sqrt{bc+a^2}}+\frac{c}{\sqrt{ca+b^2}}\geq\frac{3}{\sqrt[3]{4}}$$

122, Let $a, b, c\geq 1$, show that

$$\left(a+\frac{bc}{a^2}\right)\left(b+\frac{ca}{b^2}\right)\left(c+\frac{ab}{c^2}\right)\geq 27\cdot\sqrt[3]{(a-1)(b-1)(c-1)}$$

123, For positives a, b, c, prove that

$$\frac{a^{11}}{bc}+\frac{b^{11}}{ca}+\frac{c^{11}}{ab}+\frac{3}{a^2b^2c^2}\geq\frac{a^6+b^6+c^6+9}{2}$$

124, $x, y, z\in\left[0,\frac{1}{2}\right]$, show that:

$$\frac{x}{1+y^2}+\frac{y}{1+z^2}+\frac{z}{1+x^2}\leq\frac{6}{5}$$

125, For positives a, b, c, prove that

$$a^3+b^3+c^3+3abc+12\geq 6(a+b+c)$$

126, For positives a, b, c satisfying $a+b+c=2$, prove that

$$(1-ab)(1-bc)(1-ca)\geq (1-a^2)(1-b^2)(1-c^2)$$

127, For positives a, b, c, prove that

$$\left(1+\frac{4a}{a+b}\right)\left(1+\frac{4b}{b+c}\right)\left(1+\frac{4c}{c+a}\right)\leq 27$$

128, For positives a, b, c, prove that

$$(a^2+b^2+c^2)(ab+bc+ca)^2\geq\frac{27}{64}(a+b)^2(b+c)^2(c+a)^2$$

129, If A, B, C are the three angles of a triangle satisfying $5\cos A + 6\cos B + 7\cos C = 9$, show that we have:

$$\left(\sin\frac{A}{2}\right)^2 + \left(\sin\frac{B}{2}\right)^3 + \left(\sin\frac{C}{2}\right)^4 \geq \frac{7}{16}$$

130, Let $a, b, c \geq 0; ab + bc + ca + abc = 4$. Show that

$$\sqrt{a+3} + \sqrt{b+3} + \sqrt{c+3} \geq 6$$

131, For positives a, b, c, prove that

$$\frac{\sqrt{a^2 + 256bc}}{b+c} + \frac{\sqrt{b^2 + 256ca}}{c+a} + \frac{\sqrt{c^2 + 256ab}}{a+b} \geq 10$$

132, For positive real numbers a, b, c, show that

$$\frac{a}{b+c} + \frac{b}{c+a} + \frac{c}{a+b} \geq \frac{a}{a+b} + \frac{b}{b+c} + \frac{c}{c+a}$$

134, For positive real numbers a, b, c, prove that

$$\frac{a^5}{a+b} + \frac{b^5}{b+c} + \frac{c^5}{c+a} \geq \frac{a^3b^2}{a+b} + \frac{b^3c^2}{b+c} + \frac{c^3a^2}{c+a}$$

135, For positive real numbers a, b, c satisfying $ab + bc + ca = 1$, show that

$$a^3 + b^3 + c^3 + 3abc \geq 2abc(a+b+c)^2$$

136, For positive real numbers a, b, c satisfying $abc = 1$, show that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} \geq 2\left(\frac{\sqrt{a}}{b+c} + \frac{\sqrt{b}}{c+a} + \frac{\sqrt{c}}{a+b}\right)$$

137, For positive real numbers a, b, c satisfying $ab+bc+ca = 1$, find the minimum value of:

$$\frac{ab\sqrt{ab}}{c} + \frac{bc\sqrt{bc}}{a} + \frac{ca\sqrt{ca}}{b}$$

138, For positive real numbers a, b, c satisfying $abc = 1$, prove that:

- $\dfrac{a}{a^2+3} + \dfrac{b}{b^2+3} + \dfrac{c}{c^2+3} \leq \dfrac{3}{4}$
- $\dfrac{a}{a^3+1} + \dfrac{b}{b^3+1} + \dfrac{c}{c^3+1} \leq \dfrac{3}{2}$
- $\dfrac{a}{2a^3+1} + \dfrac{b}{2b^3+1} + \dfrac{c}{2c^3+1} \leq 1$

Also, with the same conditions, prove or disprove that:

- $\dfrac{a}{(a+3)^2} + \dfrac{b}{(b+3)^2} + \dfrac{c}{(c+3)^2} \leq \dfrac{3}{16}$

- $\left(\dfrac{2a}{a^3+1}\right)^5+\left(\dfrac{2b}{b^3+1}\right)^5+\left(\dfrac{2c}{c^3+1}\right)^5\le 3$

139, For $a,b,c>0$, prove the following inequality:

$$\frac{1}{a+b}+\frac{1}{b+c}+\frac{1}{c+a}+\frac{3}{abc}\ge\frac{12}{ab+bc+ca}$$

140, For $a,b,c\in[1,2]$, prove the following inequality:

$$2(\frac{a}{b}+\frac{b}{c}+\frac{c}{a})\ge\frac{a}{c}+\frac{c}{b}+\frac{b}{a}+3$$

141, For $a,b,c\ge 0$, prove the following inequality:

$$\frac{a^2}{b^3+2abc}+\frac{b^2}{c^3+2abc}+\frac{c^2}{a^3+2abc}\ge\frac{3\sqrt{3(a^2+b^2+c^2)}}{(a+b+c)^2}$$

142, For $a,b,c>0$, prove that

$$\frac{5a^2+2bc}{(b+c)^2}+\frac{5b^2+2ac}{(a+c)^2}+\frac{5c^2+2ab}{(a+b)^2}\ge\frac{21(a^2+b^2+c^2)}{4(ab+bc+ca)}$$

143, For $a,b,c>0$ satisfying $ab+bc+ca=2$, prove that

$$\sqrt{a^2+b^2+c^2}+2\ge a\sqrt{b^2+c^2}+b\sqrt{c^2+a^2}+c\sqrt{a^2+b^2}$$

144, For $a,b,c>0$, prove that

$$\frac{a^2+b^2}{(a+b)^2}+\frac{b^2+c^2}{(b+c)^2}+\frac{c^2+a^2}{(c+a)^2}+\frac{a+b+c}{\sqrt{3(a^2+b^2+c^2)}}\ge\frac{5}{2}$$

145, For $a,b,c>0$, prove that

$$\sqrt[9]{\frac{a^3}{b+c}}+\sqrt[9]{\frac{b^3}{c+a}}+\sqrt[9]{\frac{c^3}{a+b}}\ge\frac{1}{\sqrt[9]{2}}\left(\sqrt[9]{ab}+\sqrt[9]{bc}+\sqrt[9]{ca}\right)$$

146, For $a,b,c>0$, $ab+bc+ca=1$; find P_{min} where

$$P=\frac{1}{\sqrt{a^2-ab+b^2}}+\frac{1}{\sqrt{b^2-bc+c^2}}+\frac{1}{\sqrt{c^2-ca+a^2}}$$

147, For $a,b,c>0$, $ab+bc+ca\ge 11$; find P_{min} where

$$P\sqrt{a^2+3}+\frac{\sqrt{7}}{5}\sqrt{b^2+3}+\frac{\sqrt{3}}{5}\sqrt{c^2+3}$$

148, Let a,b,c be positive reals satisfying $ab+bc+ca=3$, Prove that:

$$\frac{1}{a^2b^3}+\frac{1}{b^2c^3}+\frac{1}{c^2a^3}+\frac{1}{3a^2-2ab+b^2}+\frac{1}{3b^2-2bc+c^2}+\frac{1}{3c^2-2ca+a^2}\ge\frac{9}{2}$$

149, For $a,b,c\geq 0$, prove that we always have:

$$\bullet\frac{a^2+b^2+c^2}{ab+bc+ca}+15\sqrt{\frac{a^3b+b^3c+c^3a}{a^2b^2+b^2c^2+c^2a^2}}\geq\frac{47}{4}.$$

$$\bullet\sqrt{\frac{a^2+b^2+c^2}{ab+bc+ca}}+\sqrt{\frac{3(a^3b+b^3c+c^3a)}{a^2b^2+b^2c^2+c^2a^2}}\geq 1+\sqrt{3}.$$

150, $a,b,c>0; a+b+c=3$, Prove that:

$$\frac{1}{\sqrt{2a^2+1}}+\frac{1}{\sqrt{2b^2+1}}+\frac{1}{\sqrt{2c^2+1}}\geq\sqrt{3}$$

151, $a,b,c\geq 0; ab+bc+ca=1$. Show that

$$\sqrt{a^2b+b^2c+c^2a}+\sqrt{ab^2+bc^2+ca^2}+3\sqrt{abc}\geq 2$$

152, $a,b,c>0; ab+bc+ca=3$. Show that

$$(a+b^2)(b+c^2)(c+a^2)\geq 8$$

153, Prove that for all positive real numbers a,b,c we have:

$$\bullet\left(\frac{a}{b+c}\right)^3+\left(\frac{b}{c+a}\right)^3+\left(\frac{c}{a+b}\right)^3+\frac{abc}{(a+b)(b+c)(c+a)}\geq\frac{1}{2}\cdot\frac{a^2+b^2+c^2}{ab+bc+ca})^2$$

$$\bullet\left(\frac{a}{b+c}\right)^3+\left(\frac{b}{c+a}\right)^3+\left(\frac{c}{a+b}\right)^3+\frac{5abc}{(a+b)(b+c)(c+a)}\geq\frac{a^2+b^2+c^2}{ab+bc+ca}$$

154, $a,b,c\geq 0$; prove that

$$\frac{2(a^2+b^2+c^2)}{(ab+bc+ca)^2}\geq\frac{a^2+b^2}{(a^2+ab+b^2)^2}+\frac{b^2+c^2}{(b^2+bc+c^2)^2}+\frac{c^2+a^2}{(c^2+ca+a^2)^2}$$

155, $a,b,c\geq 0$; prove that

$$\frac{a+b}{\sqrt{a^2+ab+b^2}}+\frac{b+c}{\sqrt{b^2+bc+c^2}}+\frac{c+a}{\sqrt{c^2+ca+a^2}}\geq 2\sqrt{3}\cdot\left(\frac{ab+bc+ca}{a^2+b^2+c^2}\right)^{\frac{4}{9}}$$

156, $a,b,c\geq 0$; prove that

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}+\frac{9}{4}.\frac{abc}{a^3+b^3+c^3}\geq\frac{15}{4}$$

157, For $x,y,z\geq 0$, prove that we always have :

$$32xyz(x+y+z)(x^2+y^2+z^2+xy+xz+yz)\leq 9(x+y)^2(x+z)^2(y+z)^2$$

158, $a,b,c\geq 0$. Prove that:

$$\frac{11}{3}(a+b+c)\geq 8\sqrt[3]{abc}+3\sqrt[3]{\frac{a^3+b^3+c^3}{3}}$$

159, $a, b, c > 0$ satisfy $a + b + c = 3$, prove that:

$$\frac{a}{\sqrt{4b + 4c^2 + 1}} + \frac{b}{\sqrt{4c + 4a^2 + 1}} + \frac{c}{\sqrt{4a + 4b^2 + 1}} \geq 1$$

160, $a + b + c = 3; a, b, c > 0$. Show that:

$$\sqrt{a} + \sqrt{b} + \sqrt{c} + \frac{6}{ab + bc + ca} \geq 5$$

161, $ab + bc + ca = 3; a, b, c > 0$. Prove that

$$\sqrt{a^2 + a} + \sqrt{b^2 + b} + \sqrt{c^2 + c} \geq 3\sqrt{2}$$

162, $a, b, c > 0; abc = 1$, show that

$$a + b + c \geq 3\sqrt[11]{\frac{a^3 + b^3 + c^3}{3}}$$

163, Let $a, b, c > 0$ satisfy $a + b + c = 3$. Prove that

$$\sqrt[3]{a^2 + ab + bc} + \sqrt[3]{b^2 + bc + ca} + \sqrt[3]{c^2 + ca + ab} \geq \sqrt[3]{3}(ab + bc + ca)$$

164, Given $a, b, c > 0$, Prove that:

$$\frac{1}{(3a + 2b + c)^2} + \frac{1}{(3b + 2c + a)^2} + \frac{1}{(3c + 2a + b)^2} \leq \frac{9}{4(ab + bc + ca)}$$

165, Let $a, b, c > 0$. Prove that:

$$a^2 + b^2 + c^2 + \frac{\sqrt{3}(ab + ac + bc)\sqrt[3]{abc}}{\sqrt{a^2 + b^2 + c^2}} \geq 2(ab + bc + ca)$$

166, Given $a, b, c > 0$ and $abc = 1$, Prove that:

$$81(1 + a^2)(1 + b^2)(1 + c^2) \leq 8(a + b + c)^4$$

167, Given $a, b, c \geq 0$ and $a + b + c = 2$, Prove that:

$$\frac{\sqrt[4]{a^2 + 6ab + b^2}}{a + b} + \frac{\sqrt[4]{b^2 + 6bc + c^2}}{b + c} + \frac{\sqrt[4]{c^2 + 6ca + a^2}}{c + a} \geq 2 + \frac{1}{\sqrt[4]{2}}$$

168, Given $a, b, c > 0$, Prove that:

$$\frac{1}{a} + \frac{1}{b} + \frac{1}{c} + \frac{3}{2a + b} + \frac{3}{2b + c} + \frac{3}{2c + a} \geq 2\sqrt{3}\left(\frac{1}{\sqrt{a(a + 2b)}} + \frac{1}{\sqrt{b(b + 2c)}} + \frac{1}{\sqrt{c(c + 2a)}}\right)$$

169, For $a, b, c > 0$, we have:

$$\frac{(a + b)^2}{a + b + 2c} + \frac{(b + c)^2}{b + c + 2a} + \frac{(c + a)^2}{c + a + 2b} \geq \sqrt{3(a^2 + b^2 + c^2)}$$

170, For $a, b, c > 0; ab + bc + ca = 3$, prove that:

$$\frac{a^2 + bc + 4ab}{a + 8b} + \frac{b^2 + ca + 4bc}{b + 8c} + \frac{c^2 + ab + 4ca}{c + 8a} \geq 2$$

171, Given a, b, c are three real numbers satisfying $a + b + c = 3$, Prove that:

$$\frac{a^2 + b^2c^2}{(b - c)^2} + \frac{b^2 + c^2a^2}{(c - a)^2} + \frac{c^2 + a^2b^2}{(a - b)^2} \geq 5$$

172, Given $a, b, c > 0$, prove that:

$$\sqrt[4]{\frac{a^3 + b^3 + c^3}{abc}} \geq 2\sqrt[4]{3}\left(\frac{a}{a + 2b + 3c} + \frac{b}{b + 2c + 3a} + \frac{c}{c + 2a + 3b}\right)$$

173, Given $a, b, c \geq 0$, show that:

$$\frac{1}{(a + b)^2} + \frac{1}{(b + c)^2} + \frac{1}{(c + a)^2} + \frac{8(ab + bc + ca)}{(a + b + c)^4} \geq \frac{11}{4(ab + bc + ca)}$$

174, Given $a, b, c \geq 0$, prove that:

$$\frac{ab}{c} + \frac{bc}{a} + \frac{ca}{b} \geq 3 \cdot \sqrt[3]{\frac{2(a^3 + b^3 + c^3) + abc}{7}}$$

175, Let $a, b, c > 0$. Prove that

$$\frac{a}{b} + \frac{b}{c} + \frac{c}{a} + \frac{8}{3} \cdot \frac{ab + bc + ca}{a^2 + b^2 + c^2} \geq \frac{17}{3}$$

Is $k = 8/3$ the best constant? If not, find the best k such that this inequality is still satisfied.

176, Let $a, b, c > 0$. Prove that

$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a} \geq \frac{6(a^2 + b^2 + c^2) - 3(ab + bc + ca)}{a + b + c}$$

177, Given $a, b, c > 0$, show that:

$$\frac{a^2}{b} + \frac{b^2}{c} + \frac{c^2}{a} \geq 3 \cdot \sqrt{\frac{a^4 + b^4 + c^4}{a^2 + b^2 + c^2}}$$

178, $a, b, c \geq 0$; prove that

$$\frac{a}{\sqrt{b^2 + ab + 9c^2}} + \frac{b}{\sqrt{c^2 + bc + 9a^2}} + \frac{c}{\sqrt{a^2 + ca + 9b^2}} \geq \frac{3}{\sqrt{11}}$$

179, For $a, b, c > 0$ such that $a + b + c = 3$, prove the following inequality:

- $$\frac{(1 + a)^2(1 + b)^2}{1 + c^2} + \frac{(1 + b)^2(1 + c)^2}{1 + a^2} + \frac{(1 + c)^2(1 + a)^2}{1 + b^2} \geq 24$$

- $\dfrac{(b+c)^5+32}{a^3+1}+\dfrac{(c+a)^5+32}{b^3+1}+\dfrac{(a+b)^5+32}{c^3+1}\geq\ 96$

180, For $a,b,c>0$, prove the following inequality:

$$\frac{(a+b)(b+c)(c+a)}{8abc}\geq\frac{(a^2+b^2)(b^2+c^2)(c^2+a^2)}{(a^2+bc)(b^2+ca)(c^2+ab)}$$

181, For $a,b,c>0$ such that $ab+bc+ca=3$, prove the following inequality:

$$(a+2b)(b+2c)(c+2a)\geq\ 8$$

182, For $a,b,c>0$, prove the following inequality:

$$21+\frac{a^3+b^3+c^3}{abc}\geq\frac{8(a+b+c)}{\sqrt[3]{abc}}$$

183,For $a,b,c>0$, prove the following inequality:

$$33+\frac{(a+b+c)(a^2+b^2+c^2)}{abc}\geq\frac{14(a+b+c)}{\sqrt[3]{abc}}$$

184, For $a,b,c>0$ and $k\geq 3$, prove the following inequality:

$$12k-9+\frac{(a+b+c)(a^2+b^2+c^2)}{abc}\geq\frac{4k(\sqrt[k]{a^3}+\sqrt[k]{b^3}+\sqrt[k]{c^3})}{\sqrt[k]{abc}}$$

185, For $a,b,c>0$, prove the following inequality:

$$\frac{\sqrt{a^2+3bc}}{(b+c)(a+8b)}+\frac{\sqrt{b^2+3ca}}{(c+a)(b+8c)}+\frac{\sqrt{c^2+3ab}}{(a+b)(c+8a)}\geq\frac{1}{a+b+c}$$

186, For $a,b,c>0$ satisfying $ab+bc+ca=3$, prove the following:

- $\sqrt{a^2+a}+\sqrt{b^2+b}+\sqrt{c^2+c}\geq 3\sqrt{2}$
- $\sqrt[3]{5a^3+3a}+\sqrt[3]{5b^3+3b}+\sqrt[3]{5c^3+3c}\geq 6$

187, For $a,b,c>0$, prove the following inequality:

$$\frac{\sqrt{ab}}{ab+c^2}+\frac{\sqrt{bc}}{bc+a^2}+\frac{\sqrt{ca}}{ca+b^2}\geq\frac{9(a^3+b^3+c^3)}{2(a+b+c)^4}$$

188, Given $a,b,c>0$, Prove that:

$$\frac{a^2}{b}+\frac{b^2}{c}+\frac{c^2}{a}\geq\sqrt[4]{3}.\sqrt[4]{\frac{a^3+b^3+c^3}{abc}}.\sqrt{a^2+b^2+c^2}$$

199,
Given $a,b,c>0$, Prove that:

$$\frac{a}{b}+\frac{b}{c}+\frac{c}{a}\geq 3.\sqrt[4]{\frac{a^3+b^3+c^3}{3abc}}$$

200, Given $a, b, c \geq 0$. Prove that:

$$\sqrt[3]{a^2+4bc}+\sqrt[3]{b^2+4ca}+\sqrt[3]{c^2+4ab} \geq \sqrt[3]{45(ab+bc+ca)}$$

201,For positives a, b, c, prove that

$$\frac{16abc-3a^3}{(b-c)^2}+\frac{16abc-3b^3}{(c-a)^2}+\frac{16abc-3c^3}{(a-b)^2} \geq 21(a+b+c)$$

202, For positives a, b, c, prove that

$$\frac{abc}{(a+b)(b+c)(c+a)} \leq \frac{(a+b)(a+b+2c)}{(3a+3b+2c)^2} \leq \frac{1}{8}$$

203, In $\triangle ABC$ show that

$$\left(\frac{\cos\frac{A}{2}}{\tan A}\right)^2+\left(\frac{\cos\frac{B}{2}}{\tan B}\right)^2+\left(\frac{\cos\frac{C}{2}}{\tan C}\right)^2+\frac{3}{4}.\frac{\sin A+\sin B+\sin C}{\tan A+\tan B+\tan C} \geq \frac{9}{8}$$

204, $a, b, c \geq 0; a+2b+3c=4$. Prove that:

$$(a^2b+b^2c+c^2a+abc)(ab^2+bc^2+ca^2+abc) \leq 8$$

205, For positives a, b, c such that $ab+bc+ca \geq 11$; prove that

$$\sqrt[3]{a^2+3}+\frac{7}{5\sqrt[3]{14}}.\sqrt[3]{b^2+3}+\frac{\sqrt[3]{9}}{5}.\sqrt[3]{c^2+3} \geq \frac{23}{5\sqrt[3]{2}}$$

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