

CD1: THỰC HIỆN PHÉP TÍNH
DẠNG 4: TÍNH TỔNG DÃY PHÂN SỐ CÓ QUY LUẬT

A. Trắc nghiệm

Câu 1. (HSG 7 huyện Thanh Sơn, Phú Thọ 2021 - 2022)

Giá trị của biểu thức $\frac{5^2}{1.6} + \frac{5^2}{6.11} + \dots + \frac{5^2}{26.31}$ là:

- A. $\frac{26}{150}$ B. $\frac{150}{26}$ C. $\frac{31}{150}$ D. $\frac{150}{31}$

Lời giải

Chọn D

$$\begin{aligned} \text{Ta có: } & \frac{5^2}{1.6} + \frac{5^2}{6.11} + \dots + \frac{5^2}{26.31} \\ &= \frac{5.5}{1.6} + \frac{5.5}{6.11} + \dots + \frac{5.5}{26.31} \\ &= 5 \cdot \frac{5}{1.6} + \frac{5}{6.11} + \dots + \frac{5}{26.31} \\ &= 5 \cdot \frac{5}{6} - \frac{1}{6} + \frac{1}{6} - \frac{1}{11} + \dots + \frac{1}{26} - \frac{1}{31} \\ &= 5 \cdot \frac{5}{6} - \frac{1}{31} = 5 \cdot \frac{30}{31} = \frac{150}{31} \end{aligned}$$

Câu 2. (HSG 7 huyện Thanh Sơn 2022 - 2023)

Cho $A = -\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} - \frac{1}{3^5} + \dots + \frac{1}{3^{100}}$. Giá trị biểu thức $B = 4|A| + \frac{1}{3^{100}}$ là

A. 0 B. 1 C. -1 D. 3

Lời giải

Chọn B

$$\begin{aligned} A &= -\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} - \frac{1}{3^5} + \dots + \frac{1}{3^{100}} \\ \Rightarrow 3A &= -1 + \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} + \dots + \frac{1}{3^{99}} \\ \Rightarrow 3A + A &= -1 + \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} + \dots + \frac{1}{3^{99}} + \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} + \dots + \frac{1}{3^{100}} \\ \Rightarrow 4A &= \frac{1}{3^{100}} - 1 \Rightarrow A = \frac{1}{4 \cdot 3^{100}} - \frac{1}{4} \\ \text{Do } 3^{100} > 1 \Rightarrow \frac{1}{3^{100}} < 1 \Rightarrow \frac{1}{3^{100}} - 1 < 0 \Rightarrow A = \frac{1}{4 \cdot 3^{100}} - \frac{1}{4} < 0 \Rightarrow |A| = -A \end{aligned}$$

$$\begin{aligned} B &= 4|A| + \frac{1}{3^{100}} = -4A + \frac{1}{3^{100}} \\ &= -4 \cdot \frac{1}{4} + \frac{1}{3^{100}} = -1 + \frac{1}{3^{100}} = -\frac{3^{100}-1}{3^{100}} = -\frac{1}{3^{100}} + 1 + \frac{1}{3^{100}} = 1 \end{aligned}$$

Câu 3. (HSG 7 huyện Tam Nông 2022 - 2023)

Giá trị của $Q = \frac{2}{1.3} + \frac{1}{2.3} + \frac{2}{6.5} + \frac{5}{10.15} + \frac{2}{7.15} + \frac{7}{21.28} + \frac{8}{28.36} + \frac{1}{4.45} + \frac{10}{45.55}$ là

A. 1. B. $\frac{99}{100}$ C. $\frac{53}{55}$ D. $\frac{54}{55}$.

Lời giải

Chọn D

$$\begin{aligned} Q &= \frac{2}{1.3} + \frac{1}{2.3} + \frac{2}{6.5} + \frac{5}{10.15} + \frac{2}{7.15} + \frac{7}{21.28} + \frac{8}{28.36} + \frac{1}{4.45} + \frac{10}{45.55} \\ Q &= \frac{2}{1.3} + \frac{3}{6.3} + \frac{4}{6.10} + \frac{5}{10.15} + \frac{6}{21.15} + \frac{7}{21.28} + \frac{8}{28.36} + \frac{9}{36.45} + \frac{10}{45.55} \\ Q &= 1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{6} + \frac{1}{6} - \frac{1}{10} + \frac{1}{10} - \frac{1}{15} + \frac{1}{15} - \frac{1}{21} + \frac{1}{21} - \frac{1}{28} + \frac{1}{28} - \frac{1}{36} + \frac{1}{36} - \frac{1}{45} + \frac{1}{45} - \frac{1}{55} \\ Q &= 1 - \frac{1}{55} = \frac{54}{55} \end{aligned}$$

Câu 4. (HSG 7 huyện Thanh Thủy 2022 - 2023) Đề 375

Cho biểu thức $N = \frac{1}{3} - \frac{2}{3^2} + \frac{3}{3^3} - \frac{4}{3^4} + \dots + \frac{99}{3^{99}} - \frac{100}{3^{100}}$. Khẳng định đúng là

A. $N < \frac{3}{16}$ B. $N > \frac{3}{16}$ C. $\frac{3}{16} < N < \frac{3}{4}$ D. $N > \frac{3}{4}$.

Lời giải

Chọn A

$$\begin{aligned} N &= \frac{1}{3} - \frac{2}{3^2} + \frac{3}{3^3} - \frac{4}{3^4} + \dots + \frac{99}{3^{99}} - \frac{100}{3^{100}} \\ 3N &= 1 - \frac{2}{3} + \frac{3}{3^2} - \frac{4}{3^3} + \dots + \frac{99}{3^{98}} - \frac{100}{3^{99}} \\ 3N + N &= 1 - \frac{2}{3} + \frac{3}{3^2} - \frac{4}{3^3} + \dots + \frac{99}{3^{98}} - \frac{100}{3^{99}} + 1 - \frac{2}{3} + \frac{3}{3^2} - \frac{4}{3^3} + \dots + \frac{99}{3^{99}} - \frac{100}{3^{100}} \\ 4N &= 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots + \frac{1}{3^{98}} - \frac{1}{3^{99}} - \frac{100}{3^{100}} \\ \frac{4N}{3} &= \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} + \dots + \frac{1}{3^{99}} - \frac{1}{3^{100}} - \frac{100}{3^{101}} \\ 4N + \frac{4N}{3} &= 1 - \frac{100}{3^{100}} - \frac{1}{3^{100}} - \frac{100}{3^{101}} \end{aligned}$$

$$\frac{16N}{3} = 1 + \frac{100}{3^{100}} + \frac{1}{3^{100}} + \frac{100}{3^{101}}$$

$$\text{Vì } \frac{100}{3^{100}} + \frac{1}{3^{100}} + \frac{100}{3^{101}} > 0 \quad \frac{16N}{3} < 1$$

nên

$$N < \frac{3}{16}$$

Vậy

Câu 5. (HSG 7 huyện Lạng Giang 2022 - 2023)

Kết quả của phép tính $A = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \dots + \frac{1}{16}(1+2+3+\dots+16)$ là:

- A. 75 B. 86 C. 76 D. 126

Lời giải

Chọn C

$$A = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \dots + \frac{1}{16}(1+2+3+\dots+16)$$

$$A = 1 + \frac{1}{2} \times \frac{(2+1) \times 2}{2} + \frac{1}{3} \times \frac{(3+1) \times 3}{2} + \dots + \frac{1}{16} \times \frac{(16+1) \times 6}{2}$$

$$A = 1 + \frac{1}{2} \times \frac{3 \times 2}{2} + \frac{1}{3} \times \frac{4 \times 3}{2} + \dots + \frac{1}{16} \times \frac{17 \times 6}{2}$$

$$A = 1 + \frac{3}{2} + \frac{4}{2} + \dots + \frac{17}{2}$$

$$A = \frac{1}{2} [2+3+4+\dots+17]$$

$$A = \frac{1}{2} \times \frac{(17+2) \times 6}{2} = \frac{1}{2} \times \frac{19 \times 6}{2} = 19 \times 3 = 76$$

B. Tự luận

Câu 1. (HSG 7 TP Bắc Ninh năm 2022-2023)

Tính: $\frac{1}{3} + \frac{3}{7} + \frac{1}{7.2} + \frac{5}{2.13} + \frac{3}{13.4} + \frac{5}{4.21}$

Lời giải

$$\frac{1}{3} + \frac{3}{7} + \frac{1}{7.2} + \frac{5}{2.13} + \frac{3}{13.4} + \frac{5}{4.21}$$

$$= 4 \times \frac{1}{12} + \frac{3}{4} + \frac{1}{7} + \frac{5}{8 \times 3} + \frac{3}{13 \times 6} + \frac{5}{16 \times 21}$$

$$= 4 \times \frac{1}{3 \times 4} + \frac{7-4}{4.7} + \frac{8-7}{7.8} + \frac{13-8}{8.13} + \frac{16-13}{13.16} + \frac{21-16}{16.21}$$

$$= 4 \times \frac{1}{12} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{8} + \frac{1}{8} - \frac{1}{13} + \frac{1}{13} - \frac{1}{16} + \frac{1}{16} - \frac{1}{21}$$

$$= 4 \times \frac{1}{12} - \frac{1}{21} = 4 \times \frac{2}{7} = \frac{8}{7}$$

Câu 2. (HSG 7 Thái Hoa Nghệ An năm 2022-2023)

$$B = \frac{1}{1.2} + \frac{1}{2.7} + \frac{1}{5.7} + \dots + \frac{1}{50.103} + \frac{1}{103}$$

Tính giá trị của biểu thức:

Lời giải

$$\begin{aligned} B &= \frac{1}{1.2} + \frac{1}{2.7} + \frac{1}{5.7} + \dots + \frac{1}{50.103} + \frac{1}{103} \\ &= \frac{3}{2} \cdot \frac{1}{1.4} + \frac{3}{4.7} + \frac{3}{7.10} + \dots + \frac{3}{100.103} + \frac{1}{103} \\ &= \frac{3}{2} \cdot \frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \dots + \frac{1}{100} - \frac{1}{103} + \frac{1}{103} \\ &= \frac{3}{2} \cdot \frac{102}{103} = \frac{2}{3} \end{aligned}$$

Câu 3. (HSG 7 Đô Lương năm 2022-2023)

$$99 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{99}{100}$$

Chứng minh.

Lời giải

$$99 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}$$

Ta có:

$$\begin{aligned} &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{100} \\ &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{100} \\ &= \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{99}{100} \end{aligned}$$

$$99 - \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100} = \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{99}{100}$$

Vậy:

Câu 4. (HSG 7 huyện Đô Lương; trường Lí Nam Đế, huyện Hưng Hà 2022 - 2023; tỉnh Bắc Giang 2012 - 2013)

$$S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2021} - \frac{1}{2022} \quad \text{và} \quad P = \frac{1}{1012} + \frac{1}{1013} + \frac{1}{1014} + \dots + \frac{1}{2022}$$

Cho:

$$(S - P)^{2022}$$

Tính

Lời giải

$$S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2021} - \frac{1}{2022}$$

Ta có

$$\begin{aligned} &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2021} + \frac{1}{2022} - \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots + \frac{1}{2022} \\ &= 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2021} + \frac{1}{2022} - \frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots + \frac{1}{2022} \end{aligned}$$

$$= \frac{1}{1012} + \frac{1}{1013} + \frac{1}{1014} + \dots + \frac{1}{2022} = P$$

$$\vdash S - P = 0 \vdash (S - P)^{2022} = 0$$

Câu 5. (HSG 7 Hà Trung năm 2022-2023)

$$D = \frac{-1}{91} + \frac{-1}{247} + \frac{-1}{475} + \frac{-1}{775} + \frac{-1}{1147}$$

Tính giá trị các biểu thức sau:

Lời giải

$$\begin{aligned} D &= \frac{-1}{91} + \frac{-1}{247} + \frac{-1}{475} + \frac{-1}{775} + \frac{-1}{1147} \\ &= \frac{-1}{7 \cdot 13} + \frac{-1}{13 \cdot 19} + \frac{-1}{19 \cdot 25} + \frac{-1}{25 \cdot 31} + \frac{-1}{31 \cdot 37} \\ &= \frac{-1}{6} \left(\frac{1}{7 \cdot 13} + \frac{1}{13 \cdot 19} + \frac{1}{19 \cdot 25} + \frac{1}{25 \cdot 31} + \frac{1}{31 \cdot 37} \right) \\ &= \frac{-1}{6} \left(\frac{1}{67} - \frac{1}{13} + \frac{1}{13} - \frac{1}{19} + \frac{1}{19} - \frac{1}{25} + \frac{1}{25} - \frac{1}{31} + \frac{1}{31} - \frac{1}{37} \right) \\ &= \frac{-1}{6} \left(\frac{1}{67} - \frac{1}{37} \right) = \frac{-1}{6} \times \frac{30}{259} = \frac{-5}{259} \end{aligned}$$

Câu 6. (HSG 7 huyện Kim Sơn, tỉnh Ninh Bình 2022 - 2023)

$$A = \frac{1}{100} - \frac{1}{100.99} - \frac{1}{99.98} - \frac{1}{98.97} - \dots - \frac{1}{3.2} - \frac{1}{2.1}$$

Rút gọn:

Lời giải

$$\begin{aligned} A &= \frac{1}{100} - \frac{1}{100.99} - \frac{1}{99.98} - \frac{1}{98.97} - \dots - \frac{1}{3.2} - \frac{1}{2.1} \\ A &= \frac{1}{100} - \frac{1}{100.99} + \frac{1}{99.98} + \frac{1}{98.97} + \dots + \frac{1}{3.2} + \frac{1}{2.1} \\ A &= \frac{1}{100} - \frac{1}{100.99} + \frac{1}{2.3} + \dots + \frac{1}{97.98} + \frac{1}{98.99} + \frac{1}{99.100} \\ A &= \frac{1}{100} - \frac{1}{100} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{97} - \frac{1}{98} + \frac{1}{98} - \frac{1}{99} + \frac{1}{99} - \frac{1}{100} \\ A &= \frac{1}{100} - \frac{1}{100} \\ A &= \frac{1}{100} - 1 + \frac{1}{100} = \frac{2}{100} - 1 = \frac{1}{50} - 1 = -\frac{49}{50} \end{aligned}$$

Câu 7. (HSG 7 Thị trấn huyện Thanh Miện 2022 - 2023)

Thực hiện phép tính:

$$S = \frac{2}{1.4} + \frac{2}{4.7} + \frac{2}{7.10} + \dots + \frac{2}{2023.2026}$$

a)

$$P = \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2022}}{\frac{2021}{1} + \frac{2020}{2} + \frac{2019}{3} + \dots + \frac{2}{2020} + \frac{1}{2021}}$$

b)

Lời giải

$$S = \frac{2}{1.4} + \frac{2}{4.7} + \frac{2}{7.10} + \dots + \frac{2}{2023.2026}$$

a)

$$\begin{aligned} P \cdot S \cdot \frac{3}{2} &= \frac{3}{1.4} + \frac{3}{4.7} + \frac{3}{7.10} + \dots + \frac{3}{2023.2026} \\ &= 1 - \frac{1}{4} + \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \dots + \frac{1}{2023} - \frac{1}{2026} \\ &= 1 - \frac{1}{2026} = \frac{2025}{2026} \end{aligned}$$

$$P \cdot S = \frac{2025}{2026} : \frac{3}{2} = \frac{675}{1013}$$

$$P = \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2022}}{\frac{2021}{1} + \frac{2020}{2} + \frac{2019}{3} + \dots + \frac{2}{2020} + \frac{1}{2021}}$$

b)

$$= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2022}}{\frac{2022-1}{1} + \frac{2022-2}{2} + \frac{2022-3}{3} + \dots + \frac{2022-2020}{2020} + \frac{2022-2021}{2021}}$$

$$= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2022}}{\frac{2022}{1} + \frac{2022}{2} + \frac{2022}{3} + \dots + \frac{2022}{2020} + \frac{2022}{2021} - \frac{1}{1} + \frac{2}{2} + \frac{3}{3} + \dots + \frac{2020}{2020} + \frac{2021}{2021}}$$

$$= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2022}}{2022 \cdot \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2020} + \frac{1}{2021} - \frac{1}{1} + \frac{2}{2} + \frac{3}{3} + \dots + \frac{2020}{2020} + \frac{2021}{2021}}$$

$$= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2022}}{2022 + 2022 \cdot \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2020} + \frac{1}{2021} - \frac{1}{1} + \frac{2}{2} + \frac{3}{3} + \dots + \frac{2020}{2020} + \frac{2021}{2021}}$$

$$= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2022}}{1 + 2022 \cdot \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2020} + \frac{1}{2021} - \frac{1}{1} + \frac{2}{2} + \frac{3}{3} + \dots + \frac{2020}{2020} + \frac{2021}{2021}}$$

$$= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{2022}}{2022 \left(\frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{2020} + \frac{1}{2021} + \frac{1}{2022} \right)} = \frac{1}{2022}$$

Câu 8. (HSG 7 Thành phố Bắc Giang 2022 - 2023)

Chứng minh rằng $\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \dots + \frac{4047}{2023^2 \cdot 2024^2} < 1$.

Lời giải

$$\begin{aligned} & \frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \dots + \frac{4047}{2023^2 \cdot 2024^2} \\ &= \frac{2^2 - 1^2}{1^2 \cdot 2^2} + \frac{3^2 - 2^2}{2^2 \cdot 3^2} + \frac{4^2 - 3^2}{3^2 \cdot 4^2} + \dots + \frac{2024^2 - 2023^2}{2023^2 \cdot 2024^2} \\ &= \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots + \frac{1}{2023^2} - \frac{1}{2024^2} = 1 - \frac{1}{2024^2} \end{aligned}$$

Ta thấy $1 - \frac{1}{2024^2} < 1$

$$\frac{3}{1^2 \cdot 2^2} + \frac{5}{2^2 \cdot 3^2} + \frac{7}{3^2 \cdot 4^2} + \dots + \frac{4047}{2023^2 \cdot 2024^2} < 1$$

Do vậy

Câu 9. (HSG 7 TX Bỉm Sơn 2022 - 2023; huyện Trục Ninh 2018 - 2019; huyện Tam Dự 2017 - 2018)

Thực hiện phép tính $\frac{1}{99.97} - \frac{1}{97.95} - \frac{1}{95.93} - \dots - \frac{1}{5.3} - \frac{1}{3.1}$

Lời giải

$$\begin{aligned} & \frac{1}{99.97} - \frac{1}{97.95} - \frac{1}{95.93} - \dots - \frac{1}{5.3} - \frac{1}{3.1} \\ &= \frac{1}{99.97} - \frac{1}{97.95} + \frac{1}{95.93} + \dots + \frac{1}{5.3} + \frac{1}{3.1} \\ &= \frac{1}{99.97} - \frac{1}{2 \cdot 95} - \frac{1}{97} + \frac{1}{93} - \frac{1}{95} + \dots + \frac{1}{3} - \frac{1}{5} + 1 - \frac{1}{3} \\ &= \frac{1}{99.97} - \frac{1}{2 \cdot 95} - \frac{1}{97} \\ &= \frac{1}{99.97} - \frac{1}{2} \times \frac{96}{97} = \frac{1}{99.97} - \frac{48}{97} = \frac{-4751}{9603} \end{aligned}$$

Câu 10. (HSG 7 Thành phố Chí Linh 2022 - 2023)

Chứng minh $S = \frac{1}{4} + \frac{2}{4^2} + \frac{3}{4^3} + \frac{4}{4^4} + \dots + \frac{2023}{4^{2023}} < \frac{1}{2}$

Lời giải

$$S = \frac{1}{4} + \frac{2}{4^2} + \frac{3}{4^3} + \frac{4}{4^4} + \dots + \frac{2023}{4^{2023}}$$

$$4S = 1 + \frac{2}{4} + \frac{3}{4^2} + \frac{4}{4^3} + \dots + \frac{2023}{4^{2022}}$$

Nên ta có

$$4S - S = 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots + \frac{1}{4^{2022}} - \frac{2023}{4^{2023}}$$

$$3S < 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots + \frac{1}{4^{2022}}$$

$$A = 1 + \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \dots + \frac{1}{4^{2022}}$$

Đặt

$$4A = 4 + 1 + \frac{1}{4} + \frac{1}{4^2} + \dots + \frac{1}{4^{2021}}$$

$$4A - A = 4 - \frac{1}{4^{2022}}$$

$$\Rightarrow 3A < 4 \Rightarrow A < \frac{4}{3}$$

$$S < \frac{4}{9} < \frac{4}{8} = \frac{1}{2}$$

Suy ra

$$S = \frac{1}{4} + \frac{2}{4^2} + \frac{3}{4^3} + \frac{4}{4^4} + \dots + \frac{2023}{4^{2023}} < \frac{1}{2}$$

Vậy

Câu 11. (HSG 7 huyện Chương Mỹ 2022 - 2023)

Cho tổng $S = \frac{1}{10^2} + \frac{1}{11^2} + \frac{1}{12^2} + \dots + \frac{1}{2024^2} + \frac{1}{2025^2}$. So sánh S với 1.

Lời giải

$$\frac{1}{k^2} < \frac{1}{(k-1)k} = \frac{1}{k-1} - \frac{1}{k} \quad (k \in \mathbb{N}, k > 1)$$

Ta có

Thay $k = 10; 11; 12; \dots; 2025$ ta được

$$\frac{1}{10^2} < \frac{1}{9 \cdot 10}; \frac{1}{11^2} < \frac{1}{10 \cdot 11}; \frac{1}{12^2} < \frac{1}{11 \cdot 12} \dots; \frac{1}{2024^2} < \frac{1}{2023 \cdot 2024}$$

$$S < \frac{1}{9} - \frac{1}{10} + \frac{1}{10} - \frac{1}{11} + \frac{1}{11} - \frac{1}{12} + \dots + \frac{1}{2024} - \frac{1}{2025}$$

Suy ra

$$\Rightarrow S < \frac{1}{9} - \frac{1}{2025} < 1$$

Vậy $S < 1$.

Câu 12. (HSG 7 huyện Mường Lát 2022 - 2023)

Cho $A = \frac{1}{3} - \frac{2}{3^2} + \frac{3}{3^3} - \frac{4}{3^4} + \dots + \frac{99}{3^{99}} - \frac{100}{3^{100}}$. Chứng minh: $A < \frac{3}{16}$

Lời giải

$$A = \frac{1}{3} - \frac{2}{3^2} + \frac{3}{3^3} - \frac{4}{3^4} + \dots + \frac{99}{3^{99}} - \frac{100}{3^{100}}$$

Ta có:

$$3A = 1 - \frac{2}{3} + \frac{3}{3^2} - \frac{4}{3^3} + \dots + \frac{99}{3^{98}} - \frac{100}{3^{99}}$$

$$3A + A = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots + \frac{1}{3^{98}} - \frac{1}{3^{99}} - \frac{100}{3^{100}}$$

$$4A = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots + \frac{1}{3^{98}} - \frac{1}{3^{99}} - \frac{100}{3^{100}} \quad (1)$$

$$B = 1 - \frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots + \frac{1}{3^{98}} - \frac{1}{3^{99}}$$

Đặt

$$3B = 3 - 1 + \frac{1}{3} - \frac{1}{3^2} + \dots + \frac{1}{3^{97}} - \frac{1}{3^{98}}$$

$$3B + B = 3 - \frac{1}{3^{99}}$$

$$4B = 3 - \frac{1}{3^{99}} < 3 \Rightarrow B < \frac{3}{4}$$

$$(2)$$

$$4A < \frac{3}{4} - \frac{100}{3^{100}} < \frac{3}{4}$$

Từ (1) và (2) suy ra

$$A < \frac{3}{16}$$

Do đó

Câu 13. (HSG 7 huyện Tiền Hải 2022 - 2023)

Cho $M = \frac{3}{3.5} + \frac{3}{5.7} + \frac{3}{7.9} + \frac{3}{9.11} + \dots + \frac{3}{53.55}$. Tính M và so sánh M với $\frac{1}{2}$.

Lời giải

$$M = \frac{3}{3.5} + \frac{3}{5.7} + \frac{3}{7.9} + \frac{3}{9.11} + \dots + \frac{3}{53.55}$$

$$= 3 \times \left(\frac{1}{3.5} + \frac{1}{5.7} + \frac{1}{7.9} + \frac{1}{9.11} + \dots + \frac{1}{53.55} \right)$$

$$= \frac{3}{2} \times \left(\frac{2}{3.5} + \frac{2}{5.7} + \frac{2}{7.9} + \frac{2}{9.11} + \dots + \frac{2}{53.55} \right)$$

$$= \frac{3}{2} \times \left(\frac{2}{3 \cdot 5} + \frac{2}{5 \cdot 7} + \frac{2}{7 \cdot 9} + \frac{2}{9 \cdot 11} + \dots + \frac{2}{53 \cdot 55} \right)$$

$$= \frac{3}{2} \times \left(\frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \frac{1}{7} - \frac{1}{9} + \frac{1}{9} - \frac{1}{11} + \dots + \frac{1}{53} - \frac{1}{55} \right)$$

$$= \frac{3}{2} \times \left(\frac{1}{3} - \frac{1}{55} \right) = \frac{3}{2} \times \frac{52}{165} = \frac{3}{2} \times \frac{52}{165} = \frac{26}{55}$$

$$M = \frac{26}{55} < \frac{26}{52} = \frac{1}{2}$$

Ta có

Câu 14. (HSG 7 Trường THCS Đồng Xuân – Phúc Yên 2022 - 2023)

Cho $A = \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots + \frac{1}{99.100}$. Chứng minh rằng $\frac{7}{12} < A < \frac{5}{6}$

Lời giải

$$\begin{aligned} A &= \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots + \frac{1}{99.100} = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{99} - \frac{1}{100} \\ &= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \frac{1}{5} + \frac{1}{6} - \frac{1}{7} + \frac{1}{8} - \frac{1}{9} + \dots - \frac{1}{99} + \frac{1}{100} \\ &= \frac{5}{6} - \frac{1}{64} + \frac{1}{50} - \frac{1}{66} + \frac{1}{70} - \frac{1}{88} + \dots - \frac{1}{99} + \frac{1}{100} < \frac{5}{6} \end{aligned} \quad (1)$$

Mặt khác

$$\begin{aligned} A &= \frac{1}{1.2} + \frac{1}{3.4} + \frac{1}{5.6} + \dots + \frac{1}{99.100} = \frac{1}{2} + \frac{1}{12} + \frac{1}{30} + \dots + \frac{1}{9900} \\ A &= \frac{1}{2} + \frac{1}{12} + \frac{1}{30} + \dots + \frac{1}{9900} = \frac{7}{12} + \frac{1}{30} + \dots + \frac{1}{9900} > \frac{7}{12} \end{aligned} \quad (2)$$

Từ (1) và (2) ta suy ra điều phải chứng minh.

Câu 15. (HSG 7 huyện Điện Bàn 2022 - 2023)

Cho biểu thức: $P = \frac{3}{(1 \times 2)^2} + \frac{5}{(2 \times 3)^2} + \frac{7}{(3 \times 4)^2} + \dots + \frac{19}{(9 \times 10)^2}$. Hãy so sánh P với 1.

Lời giải

$$\begin{aligned} P &= \frac{3}{(1 \times 2)^2} + \frac{5}{(2 \times 3)^2} + \frac{7}{(3 \times 4)^2} + \dots + \frac{19}{(9 \times 10)^2} \\ \text{Ta có:} \\ &= \frac{3}{1.4} + \frac{5}{4.9} + \frac{7}{9.16} + \dots + \frac{19}{81.100} \\ &= 1 - \frac{1}{4} + \frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} + \dots + \frac{1}{81} - \frac{1}{100} \\ P &= 1 - \frac{1}{100} < 1 \end{aligned}$$

Vậy $P < 1$.

Câu 16. (HSG 7 huyện Quảng Xương 2022 - 2023)

Tính giá trị các biểu thức sau: $B = \frac{1}{2.4} + \frac{1}{4.6} + \dots + \frac{1}{98.100}$

Lời giải

$$B = \frac{1}{2.4} + \frac{1}{4.6} + \dots + \frac{1}{98.100}$$

$$= \frac{1}{2} \times \left(\frac{1}{2} - \frac{1}{4} + \frac{1}{4} - \frac{1}{6} + \dots + \frac{1}{98} - \frac{1}{100} \right)$$

$$= \frac{1}{2} \times \left(\frac{1}{2} - \frac{1}{100} \right) = \frac{1}{2} \times \frac{49}{100} = \frac{49}{200}$$

Câu 17. (HSG 7 Thị xã Hoàng Mai 2022 - 2023)

Tính: $\frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots + \frac{1}{1+2+\dots+2023}$

Lời giải

$$\frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots + \frac{1}{1+2+\dots+2023}$$

$$= \frac{1}{(1+3).3} + \frac{1}{(1+4).4} + \frac{1}{(1+5).5} + \dots + \frac{1}{(1+2023).2023}$$

$$= \frac{2}{3.4} + \frac{2}{4.5} + \frac{2}{5.6} + \dots + \frac{2}{2023.2024}$$

$$= 2 \left(\frac{1}{3.4} + \frac{1}{4.5} + \frac{1}{5.6} + \dots + \frac{1}{2023.2024} \right)$$

$$= 2 \left(\frac{1}{3} - \frac{1}{2024} \right) = \frac{2021}{3036}$$

Câu 18. (HSG 7 huyện Hưng Hà (đề minh họa) 2022 - 2023)

Tính $A = \frac{2.2023}{1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots + \frac{1}{1+2+\dots+2023}}$

Lời giải

$$A = \frac{2.2023}{1 + \frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \dots + \frac{1}{1+2+\dots+2023}}$$

Ta có:

$$= 1 + \frac{1}{\frac{3.2}{2}} + \frac{1}{\frac{4.3}{2}} + \frac{1}{\frac{5.4}{2}} + \dots + \frac{1}{\frac{2024.2023}{2}}$$

$$= 1 + \frac{2}{2.3} + \frac{2}{3.4} + \frac{2}{4.5} + \dots + \frac{2}{2023.2024}$$

$$= 1 + 2 \left(\frac{1}{2.3} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{2023.2024} \right)$$

$$= 1 + 2 \left(\frac{1}{2} - \frac{1}{2024} \right) = 1 + 1 - \frac{1}{1012} = 2 - \frac{1}{1012} = \frac{2023}{1012}$$

$$A = \frac{2 \cdot 2023}{2023} = 2024$$

Nên

$$A = 2024$$

Câu 19. (HSG 7 huyện Quan Sơn 2022 - 2023)

$$S = \frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \dots + \frac{1}{7^{2023}}$$

Tính bằng cách hợp lý:

Lời giải

$$S = \frac{1}{7} + \frac{1}{7^2} + \frac{1}{7^3} + \dots + \frac{1}{7^{2023}}$$

$$S = 1 - \frac{1}{7} + \frac{1}{7^2} - \frac{1}{7^3} + \frac{1}{7^4} - \dots - \frac{1}{7^{2023}}$$

$$7S = 7 - 1 + \frac{1}{7} - \frac{1}{7^2} + \frac{1}{7^3} - \dots - \frac{1}{7^{2022}}$$

$$8S = 7 - \frac{1}{7^{2023}} \Rightarrow S = \frac{7^{2024} - 1}{8 \cdot 7^{2023}}$$

$$S = \frac{7^{2024} - 1}{8 \cdot 7^{2023}}$$

Vậy

Câu 20. (HSG 7 huyện Quan Sơn 2022 - 2023)

$$C = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2021} - \frac{1}{2022}$$

Cho

$$D = \frac{1}{1012} + \frac{1}{1013} + \frac{1}{1014} + \dots + \frac{1}{2021} + \frac{1}{2022}$$

$$I = (C - D)^{2021} + 2022$$

Tính

Lời giải

$$C = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2021} - \frac{1}{2022}$$

Có

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{5} + \dots + \frac{1}{2021} - \frac{1}{2022}$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{2021} + \frac{1}{2022} - \frac{1}{2022}$$

$$= \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \dots + \frac{1}{2021} + \frac{1}{2022} - \frac{1}{2022}$$

$$= \frac{1}{1012} + \frac{1}{1013} + \frac{1}{1014} + \dots + \frac{1}{2021} + \frac{1}{2022} = D$$

Suy ra $C = D$.

$$I = (C - D)^{2021} + 2022 = 2022$$

Vậy

Câu 21. (HSG 7 huyện Nông Công 2022 - 2023)

$$C = \frac{1}{5 \cdot 4} + \frac{1}{9 \cdot 14} + \frac{1}{14 \cdot 19} + \dots + \frac{1}{44 \cdot 49}$$

Tính:

Lời giải

$$C = \frac{1}{5 \cdot 4} + \frac{1}{9 \cdot 14} + \frac{1}{14 \cdot 19} + \dots + \frac{1}{44 \cdot 49}$$

$$C = \frac{1}{5 \cdot 4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{14} + \frac{1}{14} - \frac{1}{19} + \dots + \frac{1}{44} - \frac{1}{49} = \frac{1}{5} - \frac{1}{49}$$

$$C = \frac{1}{5} - \frac{1}{49} = \frac{44}{245} = \frac{4}{28}$$

$$C = \frac{4}{28}$$

Vậy

Câu 22. (HSG 7 tỉnh Thái Bình 2022 - 2023)

$$A = \frac{2}{5 \cdot 4} + \frac{2}{9 \cdot 14} + \frac{2}{14 \cdot 19} + \dots + \frac{2}{44 \cdot 49}$$

Tính:

Lời giải

$$A = \frac{2}{5 \cdot 4} + \frac{2}{9 \cdot 14} + \frac{2}{14 \cdot 19} + \dots + \frac{2}{44 \cdot 49}$$

$$A = \frac{2}{5 \cdot 4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{14} + \frac{1}{14} - \frac{1}{19} + \dots + \frac{1}{44} - \frac{1}{49} = \frac{2}{5} - \frac{1}{49}$$

$$A = \frac{2}{5} - \frac{1}{49} = \frac{94}{245} = \frac{14}{35}$$

$$A = \frac{14}{35}$$

Vậy

Câu 23. (HSG 7 trường THCS Lê Quý Đôn – Hà Đông 2022 - 2023)

$$B = \frac{1}{2 \cdot 15} + \frac{1}{15 \cdot 3} + \frac{1}{3 \cdot 21} + \dots + \frac{6}{87 \cdot 90}$$

Thực hiện phép tính:

Lời giải

$$B = \frac{6}{12 \cdot 15} + \frac{6}{15 \cdot 18} + \frac{6}{18 \cdot 21} + \dots + \frac{6}{87 \cdot 90} = 2 \left(\frac{1}{12 \cdot 15} + \frac{1}{15 \cdot 18} + \dots + \frac{1}{87 \cdot 90} \right)$$

$$B = 2 \left(\frac{1}{12} - \frac{1}{15} + \frac{1}{15} - \frac{1}{18} + \dots + \frac{1}{87} - \frac{1}{90} \right) = 2 \left(\frac{1}{12} - \frac{1}{90} \right) = \frac{13}{90}$$

$$B = \frac{13}{90}$$

Vậy

Câu 24. (HSG 7 trường Phạm Đôn Lễ - huyện Hưng Hà 2022 - 2023)

Tính

$$A = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{2022}(1+2+3+\dots+2022)$$

Lời giải

$$A = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{2022}(1+2+3+\dots+2022)$$

$$A = 1 + \frac{1}{2} \times \frac{(1+2).2}{2} + \frac{1}{3} \times \frac{(1+3).3}{2} + \frac{1}{4} \times \frac{(1+4).4}{2} + \dots + \frac{1}{2022} \times \frac{(1+2022).2022}{2}$$

$$A = 1 + \frac{1}{2} \times \frac{3.2}{2} + \frac{1}{3} \times \frac{4.3}{2} + \frac{1}{4} \times \frac{5.4}{2} + \dots + \frac{1}{2022} \times \frac{2023.2022}{2}$$

$$A = \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \dots + \frac{2023}{2}$$

$$A = \frac{2+3+4+5+\dots+2023}{2}$$

$$A = \frac{(2+2023).2022}{4} = 1023637,5$$

Vậy $A = 1023637,5$

Câu 25. (HSG 7 trường Trần Thái Tông, Hưng Hà 2022 - 2023; huyện Hà Trung 2021 - 2022)

$$A = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{20}(1+2+3+\dots+20)$$

Tính:

Lời giải

$$A = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{20}(1+2+3+\dots+20)$$

$$A = 1 + \frac{1}{2} \times \frac{(2+1).2}{2} + \frac{1}{3} \times \frac{(3+1).3}{2} + \frac{1}{4} \times \frac{(4+1).4}{2} + \dots + \frac{1}{20} \times \frac{(20+1).20}{2}$$

$$A = 1 + \frac{1}{2} \times \frac{2.3}{2} + \frac{1}{3} \times \frac{3.4}{2} + \frac{1}{4} \times \frac{4.5}{2} + \dots + \frac{1}{20} \times \frac{20.21}{2}$$

$$A = \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \dots + \frac{21}{2}$$

$$A = \frac{1}{2} \cdot (2+3+4+5+\dots+21)$$

$$= \frac{1}{2} \cdot \frac{(21+2).20}{2} = \frac{1}{2} \cdot \frac{20.23}{2} = 115$$

Câu 26. (HSG 7 huyện Hưng Hà, tỉnh, trường Nguyễn Tông Quai 2022 - 2023)

$$B = \frac{10}{7.12} + \frac{10}{12.17} + \frac{10}{17.22} + \dots + \frac{10}{2012.2017} + \frac{10}{2017.2022}$$

Thực hiện phép tính :

Lời giải

$$\begin{aligned}
 B &= \frac{10}{7.12} + \frac{10}{12.17} + \frac{10}{17.22} + \dots + \frac{10}{2012.2017} + \frac{10}{2017.2022} \\
 &= 2 \cdot \frac{5}{7.12} + \frac{5}{12.17} + \frac{5}{17.22} + \dots + \frac{5}{2012.2017} + \frac{5}{2017.2022} \\
 &= 2 \cdot \frac{1}{7} - \frac{1}{12} + \frac{1}{12} - \frac{1}{17} + \frac{1}{17} - \frac{1}{22} + \dots + \frac{1}{2012} - \frac{1}{2017} + \frac{1}{2017} - \frac{1}{2022} \\
 &= 2 \cdot \frac{1}{7} - \frac{1}{2022} = 2 \cdot \frac{2022-7}{2022 \cdot 7} = \frac{2015}{7077} \\
 B &= \frac{2015}{7077}
 \end{aligned}$$

Vậy

Câu 27. (HSG 7 huyện Hưng Hà, tỉnh, trường Lưu Khánh Đàm 2022 - 2023)

Thực hiện phép tính :

$$\frac{3}{(1.2)^2} + \frac{5}{(2.3)^2} + \frac{7}{(3.4)^2} + \dots + \frac{19}{(9.10)^2}$$

Lời giải

$$\begin{aligned}
 &\frac{3}{(1.2)^2} + \frac{5}{(2.3)^2} + \frac{7}{(3.4)^2} + \dots + \frac{19}{(9.10)^2} \\
 &= \frac{3}{1.4} + \frac{5}{4.9} + \frac{7}{9.16} + \dots + \frac{19}{81.100} \\
 &= \frac{1}{1} - \frac{1}{4} + \frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{16} + \dots + \frac{1}{81} - \frac{1}{100} \\
 &= 1 - \frac{1}{100} = \frac{99}{100}
 \end{aligned}$$

Câu 28. (HSG 7 2022 - 2023)

Tính giá trị của biểu thức sau:

$$C = 2023 - \frac{1}{2}(1+2) - \frac{1}{3}(1+2+3) - \frac{1}{4}(1+2+3+4) - \dots - \frac{1}{2022}(1+2+3+\dots+2022)$$

Lời giải

$$\begin{aligned}
 C &= 2023 - \frac{1}{2}(1+2) - \frac{1}{3}(1+2+3) - \frac{1}{4}(1+2+3+4) - \dots - \frac{1}{2022}(1+2+3+\dots+2022) \\
 &= 2023 - \frac{1}{2} \cdot \frac{(1+2) \cdot 2}{2} - \frac{1}{3} \cdot \frac{(1+3) \cdot 3}{2} - \frac{1}{4} \cdot \frac{(1+4) \cdot 4}{2} - \dots - \frac{1}{2022} \cdot \frac{(1+2022) \cdot 2022}{2} \\
 &= 2023 - \frac{3}{2} - \frac{4}{3} - \frac{5}{2} - \dots - \frac{2023}{2} \\
 &= 2023 - \frac{3+4+5+\dots+2023}{2} = 2023 - \frac{(3+2023) \cdot 2021}{4} = \frac{-4086454}{4}
 \end{aligned}$$

Câu 29. (HSG 7 huyện Hưng Hà, tỉnh, trường Duyên Hải 2022 - 2023)

Tính giá trị của biểu thức

$$B = 1 + (1+2) + (1+2+3) + (1+2+3+4) + \frac{1}{4} + (1+2+3+\frac{1}{4}+16)$$

Lời giải

Áp dụng công thức tính tổng các số hạng của dãy số cách đều

$$1 + 2 + 3 + \dots + n = \frac{(1+n)n}{2} \quad \text{ta có:}$$

$$1 + 2 = \frac{(1+2)2}{2}; \quad 1 + 2 + 3 = \frac{(1+3)3}{2}; \quad 1 + 2 + 3 + 4 = \frac{(1+4)4}{2}; \dots;$$

$$1 + 2 + 3 + \dots + 16 = \frac{(1+16)16}{2}$$

$$B = 1 + \frac{1}{2} \times \frac{(1+2)2}{2} + \frac{1}{3} \times \frac{(1+3)3}{2} + \frac{1}{4} \times \frac{(1+4)4}{2} + \dots + \frac{1}{16} \times \frac{(1+16)16}{2}$$

Khi đó:

$$B = \frac{2 + 3 + 4 + 5 + \dots + 17}{2}$$

$$B = \frac{(2+17) \cdot 16}{2}$$

(Áp dụng công thức tính tổng các số hạng của dãy số cách đều)

$$B = 152$$

$$\text{Vậy } B = 152$$

Câu 30. (HSG 7 huyện Hưng Hà, tỉnh, trường Văn Lang 2022 - 2023)

$$M = 2 - \frac{5}{3} + \frac{7}{6} - \frac{9}{10} + \frac{11}{15} - \frac{13}{21} + \frac{15}{28} - \frac{17}{36} + \frac{19}{45}$$

Tính giá trị biểu thức

Lời giải

$$M = 2 - \frac{5}{3} + \frac{7}{6} - \frac{9}{10} + \frac{11}{15} - \frac{13}{21} + \frac{15}{28} - \frac{17}{36} + \frac{19}{45}$$

$$= 2 - \frac{5}{6} + \frac{7}{12} - \frac{9}{20} + \frac{11}{30} - \frac{13}{42} + \frac{15}{56} - \frac{17}{72} + \frac{19}{90}$$

$$= 2 - \frac{5}{12} + \frac{1}{2} - \frac{1}{2} + \frac{1}{3} - \frac{1}{3} + \frac{1}{4} - \dots - \frac{1}{8} + \frac{1}{9} + \frac{1}{9} - \frac{1}{10} = 2 - \frac{5}{12} + \frac{1}{10} = \frac{6}{5}$$

Câu 31. (HSG 7 tỉnh Ninh Bình; trường Vũ Thị Thục, Hưng Hà 2022 - 2023)

$$A = -\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} - \frac{1}{3^5} + \dots + \frac{1}{3^{100}}$$

Cho biểu thức

$$B = 4|A| + \frac{1}{3^{100}}$$

Tính giá trị biểu thức

Lời giải

$$A = -\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} - \frac{1}{3^5} + \dots + \frac{1}{3^{100}}$$

$$3A = -1 + \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} + \dots + \frac{1}{3^{99}}$$

$$3A + A = \left(1 + \frac{1}{3} - \frac{1}{3^2} + \frac{1}{3^3} - \frac{1}{3^4} + \dots + \frac{1}{3^{99}}\right) + \left(\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \frac{1}{3^4} - \frac{1}{3^5} + \dots + \frac{1}{3^{100}}\right)$$

$$4A = \frac{1}{3^{100}} - 1 \Rightarrow A = \frac{1}{4 \cdot 3^{100}} - \frac{1}{4}$$

$$\text{Do } 3^{100} > 1 \Rightarrow \frac{1}{3^{100}} < 1 \Rightarrow \frac{1}{3^{100}} - 1 < 0 \Rightarrow A = \frac{1}{4 \cdot 3^{100}} - \frac{1}{4} < 0$$

$$\text{Khi đó: } |A| = -A$$

$$\Rightarrow B = 4|A| + \frac{1}{3^{100}} = -4A + \frac{1}{3^{100}}$$

$$= -4 \cdot \left(\frac{1}{4 \cdot 3^{100}} - \frac{1}{4}\right) + \frac{1}{3^{100}} = -\frac{1}{3^{100}} + 1 + \frac{1}{3^{100}} = 1$$

Câu 32. (HSG 7 huyện Nho Quan 2022 - 2023)

$$A = \frac{5^2}{1.6} + \frac{5^2}{6.11} + \frac{5^2}{11.16} + \dots + \frac{5^2}{56.61} + \frac{5^2}{61.66}$$

Tính giá trị biểu thức

Lời giải

$$A = \frac{5^2}{1.6} + \frac{5^2}{6.11} + \frac{5^2}{11.16} + \dots + \frac{5^2}{56.61} + \frac{5^2}{61.66}$$

$$= 5 \left(\frac{5}{1.6} + \frac{5}{6.11} + \frac{5}{11.16} + \dots + \frac{5}{56.61} + \frac{5}{61.66} \right)$$

$$= 5 \left(\frac{1}{6} - \frac{1}{6} + \frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \dots + \frac{1}{56} - \frac{1}{61} + \frac{1}{61} - \frac{1}{66} \right)$$

$$= 5 \left(\frac{1}{66} \right) = 5 \times \frac{65}{66} = \frac{325}{66}$$

$$A = \frac{325}{66}$$

Vậy

Câu 33. (HSG 7 huyện Thanh Miện, 2021 - 2022)

$$A = \frac{1}{12} + \frac{7}{12.17} + \frac{7}{17.22} + \frac{7}{22.27} + \dots + \frac{7}{2017.2022}$$

Thực hiện phép tính:

Lời giải

$$A = \frac{1}{12} + \frac{7}{12.17} + \frac{7}{17.22} + \frac{7}{22.27} + \dots + \frac{7}{2017.2022}$$

$$= \frac{7}{7.12} + \frac{7}{12.17} + \frac{7}{17.22} + \frac{7}{22.27} + \dots + \frac{7}{2017.2022}$$

$$A = \frac{7}{5} \left(\frac{5}{7.12} + \frac{5}{12.17} + \frac{5}{17.22} + \frac{5}{22.27} + \dots + \frac{5}{2017.2022} \right)$$

$$A = \frac{7}{5} \left(\frac{1}{12} - \frac{1}{12} + \frac{1}{12} - \frac{1}{17} + \frac{1}{17} - \frac{1}{22} + \dots + \frac{1}{2017} - \frac{1}{2022} \right)$$

$$A = \frac{7}{5} - \frac{1}{2022} = \frac{7}{5} \cdot \frac{2015}{7 \cdot 2022} = \frac{403}{2022}$$

Câu 34. (HSG 7 huyện Nghĩa Hành, 2021 - 2022)

$$P = \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6} + \dots + \frac{1}{2019.2020.2021} + \frac{1}{2020.2021.2022}$$

Thực hiện phép tính:

Lời giải

$$P = \frac{1}{2.3.4} + \frac{1}{3.4.5} + \frac{1}{4.5.6} + \dots + \frac{1}{2019.2020.2021} + \frac{1}{2020.2021.2022}$$

$$P = \frac{1}{2} \cdot \frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \dots + \frac{1}{2020.2021} - \frac{1}{2021.2022}$$

$$P = \frac{1}{2} \cdot \frac{1}{2.3} - \frac{1}{2021.2022}$$

$$P = \frac{1}{2} \cdot \frac{337.2021 - 1}{2021.2022} = \frac{340538}{2021.2022} = \frac{170269}{2021.1011} = \frac{170269}{2042220}$$

Câu 35. (HSG 7 huyện Cao Lộc, 2021 - 2022)

$$A = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{2019.2021} + \frac{1}{2021.2023}$$

Thực hiện phép tính:

Lời giải

$$A = \frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \dots + \frac{1}{2019.2021} + \frac{1}{2021.2023}$$

$$= \frac{1}{2} \cdot \frac{1}{1.3} + \frac{2}{3.5} + \frac{2}{5.7} + \dots + \frac{2}{2019.2021} + \frac{2}{2021.2023}$$

$$= \frac{1}{2} \cdot \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \dots + \frac{1}{2019} - \frac{1}{2021} + \frac{1}{2021} - \frac{1}{2023}$$

$$= \frac{1}{2} \cdot \frac{1}{1} - \frac{1}{2023} = \frac{1}{2} \cdot \frac{2022}{2023} = \frac{1011}{2023}$$

Câu 36. (HSG 7 Thành phố Vũng Tàu, 2021 - 2022)

$$A = \frac{5}{28} + \frac{5}{70} + \frac{5}{130} + \frac{5}{208} + \frac{1}{4} + \frac{5}{9700}$$

Thực hiện phép tính:

Lời giải

$$A = \frac{5}{28} + \frac{5}{70} + \frac{5}{130} + \frac{5}{208} + \dots + \frac{5}{9700}$$

$$= \frac{5}{3} \cdot \frac{3}{4.7} + \frac{5}{3} \cdot \frac{3}{7.10} + \frac{5}{3} \cdot \frac{3}{10.13} + \frac{5}{3} \cdot \frac{3}{13.16} + \dots + \frac{5}{3} \cdot \frac{3}{97.100}$$

$$= \frac{5}{3} \cdot \frac{1}{4} - \frac{1}{7} + \frac{1}{7} - \frac{1}{10} + \frac{1}{10} - \frac{1}{13} + \frac{1}{13} - \frac{1}{16} + \dots + \frac{1}{97} - \frac{1}{100} = \frac{5}{3} \cdot \frac{1}{4} - \frac{1}{100} = \frac{5}{3} \cdot \frac{24}{100} = \frac{2}{5}$$

Câu 37. (HSG 7 huyện Cửa Lò, tỉnh Nghệ An, 2020 - 2021)

Thực hiện phép tính:

$$A = \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90}$$

Lời giải

$$\begin{aligned} A &= \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} \\ &= \frac{1}{2} + \frac{1}{6} + \frac{1}{12} + \frac{1}{20} + \frac{1}{30} + \frac{1}{42} + \frac{1}{56} + \frac{1}{72} + \frac{1}{90} \\ &= \frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \frac{1}{3 \cdot 4} + \frac{1}{4 \cdot 5} + \frac{1}{5 \cdot 6} + \frac{1}{6 \cdot 7} + \frac{1}{7 \cdot 8} + \frac{1}{8 \cdot 9} + \frac{1}{9 \cdot 10} \\ &= \frac{1}{1} - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \frac{1}{5} - \frac{1}{6} + \frac{1}{6} - \frac{1}{7} + \frac{1}{7} - \frac{1}{8} + \frac{1}{8} - \frac{1}{9} + \frac{1}{9} - \frac{1}{10} \\ &= \frac{1}{1} - \frac{1}{10} = \frac{9}{10} \end{aligned}$$

Câu 38. (HSG 7 huyện Bát Xát, tỉnh Lào Cai, 2021 - 2022)

$$M = \frac{1}{2.6} + \frac{1}{3.8} + \frac{1}{4.10} + \dots + \frac{1}{498.998} + \frac{1}{499.1000}$$

Thực hiện phép tính:

Lời giải

$$\begin{aligned} M &= \frac{1}{2.6} + \frac{1}{3.8} + \frac{1}{4.10} + \dots + \frac{1}{498.998} + \frac{1}{499.1000} \\ &= \frac{2}{4.6} + \frac{2}{6.8} + \frac{2}{8.10} + \dots + \frac{2}{996.998} + \frac{2}{998.1000} \\ &= \frac{1}{4} - \frac{1}{6} + \frac{1}{6} - \frac{1}{8} + \frac{1}{8} - \frac{1}{10} + \dots + \frac{1}{996} - \frac{1}{998} + \frac{1}{998} - \frac{1}{1000} = \frac{1}{4} - \frac{1}{1000} = \frac{249}{1000} \end{aligned}$$

Câu 39. (HSG 7 huyện Mường La, 2021 - 2022)

$$B = \frac{13}{4.9} - \frac{23}{9.14} + \frac{33}{14.19} - \frac{43}{19.24} + \frac{1}{4} + \frac{95}{44.49} - \frac{19.54}{49.54}$$

Thực hiện phép tính:

Lời giải

$$\begin{aligned} B &= \frac{13}{4.9} - \frac{23}{9.14} + \frac{33}{14.19} - \frac{43}{19.24} + \frac{1}{4} + \frac{95}{44.49} - \frac{103}{49.54} \\ &= \frac{4+9}{4.9} - \frac{9+14}{9.14} + \frac{14+19}{14.19} - \frac{19+24}{19.24} + \frac{1}{4} + \frac{44+49}{44.49} - \frac{49+54}{49.54} \\ &= \frac{1}{4} + \frac{1}{9} - \frac{1}{9} - \frac{1}{14} + \frac{1}{14} + \frac{1}{19} - \frac{1}{19} - \frac{1}{24} + \frac{1}{4} + \frac{1}{44} + \frac{1}{49} - \frac{1}{49} - \frac{1}{54} = \frac{1}{4} - \frac{1}{54} = \frac{25}{108} \end{aligned}$$

Câu 40. (HSG 7 huyện Quảng Trạch, tỉnh Quảng Bình, 2021 - 2022)

Chứng minh rằng với $n \in \mathbb{N}, n \geq 3$ ta có: $A = \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \dots + \frac{1}{n^3} < \frac{1}{12}$.

Lời giải

Với $n \in \mathbb{N}, n \geq 3$ ta có: $A = \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \dots + \frac{1}{n^3}$

$$2A = \frac{2}{3^3} + \frac{2}{4^3} + \frac{2}{5^3} + \dots + \frac{2}{n^3}$$

Ta có: $\frac{2}{3^3} < \frac{2}{2.3.4} = \frac{1}{2.3} - \frac{1}{3.4}$

$$\frac{2}{4^3} < \frac{2}{3.4.5} = \frac{1}{3.4} - \frac{1}{4.5}$$

$$\frac{2}{5^3} < \frac{2}{4.5.6} = \frac{1}{4.5} - \frac{1}{5.6}$$

.....

$$\frac{2}{n^3} < \frac{2}{(n-1)n(n+1)} = \frac{1}{(n-1)n} - \frac{1}{n(n+1)}$$

$$2A = \frac{2}{3^3} + \frac{2}{4^3} + \frac{2}{5^3} + \dots + \frac{2}{n^3} < \frac{1}{2.3} - \frac{1}{3.4} + \frac{1}{3.4} - \frac{1}{4.5} + \frac{1}{4.5} - \frac{1}{5.6} + \dots + \frac{1}{(n-1).n} - \frac{1}{n(n+1)}$$

$$2A < \frac{1}{2.3} - \frac{1}{n(n+1)}$$

$$A < \frac{1}{12} - \frac{1}{2n(n+1)} < \frac{1}{12}$$

Vậy $A = \frac{1}{3^3} + \frac{1}{4^3} + \frac{1}{5^3} + \dots + \frac{1}{n^3} < \frac{1}{12}$ (với $n \in \mathbb{N}, n \geq 3$).

Câu 41. (HSG 7 huyện Đức Thọ, tỉnh Hà Tĩnh, 2021 - 2022)

$$Q = \frac{3}{1.2} - \frac{5}{2.3} + \frac{7}{3.4} - \dots + \frac{2019}{1009.1010} - \frac{2021}{1010.1011}$$

Tính giá trị của biểu thức

Lời giải

Ta có $Q = \frac{3}{1.2} - \frac{5}{2.3} + \frac{7}{3.4} - \dots + \frac{2019}{1009.1010} - \frac{2021}{1010.1011}$

$$= \frac{1+2}{1.2} - \frac{2+3}{2.3} + \frac{3+4}{3.4} - \dots + \frac{1009+1010}{1009.1010} - \frac{1010+1011}{1010.1011}$$

$$= \frac{1}{1} + \frac{1}{2} - \frac{1}{2} - \frac{1}{3} + \frac{1}{3} + \frac{1}{4} - \dots + \frac{1}{1009} + \frac{1}{1010} - \frac{1}{1010} - \frac{1}{1011}$$

$$= 1 + \frac{1}{2} - \frac{1}{2} - \frac{1}{3} + \frac{1}{3} + \frac{1}{4} - \dots + \frac{1}{1009} + \frac{1}{1010} - \frac{1}{1010} - \frac{1}{1011}$$

$$= 1 - \frac{1}{1011} = \frac{1010}{1011}$$

Câu 42. (HSG 7 huyện 2016 - 2017)

$$\frac{1}{4.9} + \frac{1}{9.14} + \frac{1}{14.19} + \dots + \frac{1}{44.49}$$

Thực hiện phép tính:

Lời giải

$$\begin{aligned}
 & \frac{1}{4.9} + \frac{1}{9.14} + \frac{1}{14.19} + \dots + \frac{1}{44.49} \\
 &= \frac{1}{5} \left(\frac{1}{4} - \frac{1}{9} + \frac{1}{9} - \frac{1}{14} + \frac{1}{14} - \frac{1}{19} + \dots + \frac{1}{44} - \frac{1}{49} \right) \\
 &= \frac{1}{5} \left(\frac{1}{4} - \frac{1}{49} \right) = \frac{1}{5} \cdot \frac{45}{196} = \frac{9}{196}
 \end{aligned}$$

Câu 43. (HSG 7 huyện Hương Khê 2021 - 2022)

Tính $E = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{100}(1+1+2+\dots+100)$

Lời giải

$$\begin{aligned}
 E &= 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{100}(1+1+2+\dots+100) \\
 &= 1 + \frac{1}{2} \cdot \frac{2.3}{2} + \frac{1}{3} \cdot \frac{3.4}{2} + \frac{1}{4} \cdot \frac{4.5}{2} + \dots + \frac{1}{100} \cdot \frac{100.101}{2} \\
 &= 1 + \frac{3}{2} + \frac{4}{3} + \frac{5}{2} + \dots + \frac{101}{2} = \frac{1}{2} + \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \dots + \frac{101}{2} - \frac{1}{2} \\
 &= \frac{1}{2}(1+2+3+\dots+101) - \frac{1}{2} = 2575
 \end{aligned}$$

Vậy $E = 2575$

Câu 44. (HSG 7 Lê Hồng Phong 2018 - 2019)

Tính nhanh : $N = \frac{1}{1000} - \frac{1}{1000.999} - \frac{1}{999.998} - \frac{1}{998.997} - \dots - \frac{1}{3.2} - \frac{1}{2.1}$

Lời giải

$$\begin{aligned}
 N &= \frac{1}{1000} - \frac{1}{1000.999} + \frac{1}{999.998} - \frac{1}{998.997} + \dots + \frac{1}{3.2} + \frac{1}{2.1} \\
 &= \frac{1}{1000} - \frac{1}{999} + \frac{1}{1000} - \frac{1}{998} + \frac{1}{999} - \frac{1}{997} + \frac{1}{998} - \frac{1}{996} + \dots + \frac{1}{2} - \frac{1}{3} + 1 - \frac{1}{2} \\
 &= \frac{1}{1000} - \frac{1}{1000} + \frac{499}{500} = \frac{499}{500}
 \end{aligned}$$

Câu 45. (HSG 7 huyện Bát Xát 2022 - 2023)

Thực hiện các phép tính: $D = \frac{5^2}{1.6} + \frac{5^2}{6.11} + \dots + \frac{5^2}{26.31}$

Lời giải

$$\begin{aligned}
 D &= \frac{5^2}{1.6} + \frac{5^2}{6.11} + \dots + \frac{5^2}{26.31} \\
 D &= 5 \left(\frac{5}{1.6} + \frac{5}{6.11} + \dots + \frac{5}{26.31} \right)
 \end{aligned}$$

$$= 5 \left(\frac{1}{6} - \frac{1}{6} + \frac{1}{6} - \frac{1}{11} + \frac{1}{11} - \frac{1}{16} + \dots + \frac{1}{26} - \frac{1}{31} \right)$$

$$= 5 \left(\frac{1}{31} - \frac{1}{31} \right) = 5 \cdot \frac{30}{31} = \frac{150}{31}$$

$$D = \frac{150}{31}$$

Vậy

Câu 46. (HSG 7 huyện Kim Thành 2022 - 2023)

$$A = \frac{1}{11 \cdot 14} + \frac{1}{14 \cdot 17} + \frac{1}{17 \cdot 20} + \frac{1}{20 \cdot 23} + \frac{1}{23 \cdot 26} + \frac{1}{26 \cdot 29} + \frac{1}{29 \cdot 32}$$

Tính nhanh giá trị của biểu thức sau

Lời giải

$$A = \frac{1}{11 \cdot 14} + \frac{1}{14 \cdot 17} + \frac{1}{17 \cdot 20} + \frac{1}{20 \cdot 23} + \frac{1}{23 \cdot 26} + \frac{1}{26 \cdot 29} + \frac{1}{29 \cdot 32}$$

$$A = \frac{1}{3} \left(\frac{3}{11 \cdot 14} + \frac{3}{14 \cdot 17} + \frac{3}{17 \cdot 20} + \frac{3}{20 \cdot 23} + \frac{3}{23 \cdot 26} + \frac{3}{26 \cdot 29} + \frac{3}{29 \cdot 32} \right)$$

$$A = \frac{1}{3} \left(\frac{3 \cdot 14 - 11}{11 \cdot 14} + \frac{3 \cdot 17 - 14}{14 \cdot 17} + \frac{3 \cdot 20 - 17}{17 \cdot 20} + \frac{3 \cdot 23 - 20}{20 \cdot 23} + \frac{3 \cdot 26 - 23}{23 \cdot 26} + \frac{3 \cdot 29 - 26}{26 \cdot 29} + \frac{3 \cdot 32 - 29}{29 \cdot 32} \right)$$

$$A = \frac{1}{3} \left(\frac{31}{11 \cdot 14} + \frac{31}{14 \cdot 17} + \frac{31}{17 \cdot 20} + \frac{31}{20 \cdot 23} + \frac{31}{23 \cdot 26} + \frac{31}{26 \cdot 29} + \frac{31}{29 \cdot 32} \right)$$

$$A = \frac{1}{3} \left(\frac{1}{11} - \frac{1}{14} + \frac{1}{14} - \frac{1}{17} + \frac{1}{17} - \frac{1}{20} + \dots + \frac{1}{29} - \frac{1}{32} \right)$$

$$A = \frac{1}{3} \left(\frac{1}{11} - \frac{1}{32} \right) = \frac{1}{3} \cdot \frac{30}{451} = \frac{10}{451}$$

$$A = \frac{10}{451}$$

Vậy

Câu 47. (HSG 7 trường Quang Trung 2018 - 2019)

$$B = \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}}{\frac{1}{99} + \frac{1}{98} + \frac{1}{97} + \dots + \frac{1}{1}}$$

Thực hiện phép tính:

Lời giải

$$B = \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}}{\frac{1}{99} + \frac{1}{98} + \frac{1}{97} + \dots + \frac{1}{1}}$$

$$= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}}{\frac{100-1}{1} + \frac{100-2}{2} + \frac{100-3}{3} + \dots + \frac{100-99}{99} + \frac{100-100}{100}}$$

$$\begin{aligned}
 &= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}}{\frac{100}{1} + \frac{100}{2} + \frac{100}{3} + \dots + \frac{100}{99}} \\
 &= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}}{100 + 100 \cdot \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{99}} \\
 &= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}}{1 + 100 \cdot \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{99}} \\
 &= \frac{\frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}}{100 \cdot \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{100}} = \frac{1}{100}
 \end{aligned}$$

Câu 48. (HSG 7 huyện Tam Nông 2018 - 2019)

Tính: $B = \frac{2}{1.3} + \frac{2}{3.5} + \dots + \frac{2}{2011.2013}$

Lời giải

$$\begin{aligned}
 B &= \frac{2}{1.3} + \frac{2}{3.5} + \dots + \frac{2}{2011.2013} \\
 &= \frac{1}{1} - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2011} - \frac{1}{2013} \\
 &= 1 - \frac{1}{2013} = \frac{2012}{2013}
 \end{aligned}$$

Câu 49. (HSG 7 huyện Hoài Nhơn 2018 - 2019)

Cho $S = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2013} - \frac{1}{2014} + \frac{1}{2015}$ và

$P = \frac{1}{1008} + \frac{1}{1009} + \frac{1}{1010} + \dots + \frac{1}{2014} + \frac{1}{2015}$. Tính $(S - P)^{2016}$

Lời giải

Ta có:

$$\begin{aligned}
 P &= \frac{1}{1008} + \frac{1}{1009} + \frac{1}{1010} + \dots + \frac{1}{2014} + \frac{1}{2015} \\
 &= \frac{1}{1008} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{1006} + \frac{1}{1007} + \frac{1}{1008} + \dots + \frac{1}{2014} + \frac{1}{2015} \\
 &= \frac{1}{1008} + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{1006} + \frac{1}{1007} + \frac{1}{1008} + \dots + \frac{1}{2014} + \frac{1}{2015} + 2\left(\frac{1}{2} + \frac{1}{4} + \frac{1}{6} + \dots + \frac{1}{2012} + \frac{1}{2014}\right) \\
 &= 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots + \frac{1}{2013} - \frac{1}{2014} + \frac{1}{2015} = S
 \end{aligned}$$

Do đó $(S - P)^{2016} = 0$

Câu 50. (HSG 7 huyện Sơn Dương 2016 - 2017)

Tính $B = \frac{1}{19} + \frac{9}{19.29} + \frac{9}{29.39} + \frac{9}{39.49} + \dots + \frac{9}{1999.2009}$

Lời giải

$$\begin{aligned} B &= \frac{1}{19} + \frac{9}{19.29} + \frac{9}{29.39} + \frac{9}{39.49} + \dots + \frac{9}{1999.2009} \\ &= \frac{9}{9.19} + \frac{9}{19.29} + \frac{9}{29.39} + \frac{9}{39.49} + \dots + \frac{9}{1999.2009} \\ &= \frac{9}{10} \cdot \frac{10}{9.19} + \frac{10}{19.29} + \frac{10}{29.39} + \frac{10}{39.49} + \dots + \frac{10}{1999.2009} \\ &= \frac{9}{10} \cdot \frac{10}{9} - \frac{1}{19} + \frac{1}{19} - \frac{1}{29} + \frac{1}{29} - \frac{1}{39} + \dots + \frac{1}{1999} - \frac{1}{2009} \\ &= \frac{9}{10} \cdot \frac{10}{9} - \frac{1}{2009} = \frac{200}{2009} \end{aligned}$$

Câu 51. (HSG 7 huyện Quế Sơn năm học 2018 - 2019; huyện Kim Sơn 2017 - 2018; trường Đức Thắng, Hiệp Hòa 2016 - 2017)

Tính: $P = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{16}(1+2+3+\dots+16)$

Lời giải

$$\begin{aligned} P &= 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{16}(1+2+3+\dots+16) \\ P &= 1 + \frac{1}{2} \cdot \frac{2.3}{2} + \frac{1}{3} \cdot \frac{3.4}{2} + \frac{1}{4} \cdot \frac{4.5}{2} + \dots + \frac{1}{16} \cdot \frac{16.17}{2} \\ P &= \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \dots + \frac{17}{2} \\ P &= \frac{1}{2}(1+2+3+\dots+17-1) = 76 \end{aligned}$$

Câu 52. (HSG 7 huyện Thanh Oai, trường THCS Hồng Dương năm học 2017 - 2018)

Tính: $B = -\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots + \frac{1}{3^{50}} - \frac{1}{3^{51}}$

Lời giải

$$\begin{aligned} B &= -\frac{1}{3} + \frac{1}{3^2} - \frac{1}{3^3} + \dots + \frac{1}{3^{50}} - \frac{1}{3^{51}} \\ B &= \frac{1}{(-3)} + \frac{1}{(-3)^2} + \frac{1}{(-3)^3} + \dots + \frac{1}{(-3)^{50}} + \frac{1}{(-3)^{51}} \\ -\frac{1}{3}B &= \frac{1}{(-3)^2} + \frac{1}{(-3)^3} + \dots + \frac{1}{(-3)^{51}} + \frac{1}{(-3)^{52}} \end{aligned}$$

$$p \frac{4}{3} B = \frac{1}{-3} - \frac{1}{(-3)^{52}} = \frac{-3^{51} - 1}{3^{52}}$$

$$p B = \frac{-3^{51} - 1}{4 \cdot 3^{51}}$$

Câu 53. (HSG 7 huyện Đức Thọ năm học 2017 - 2018)

$$\frac{1}{7 \cdot 8} + \frac{1}{8 \cdot 15} + \frac{1}{15 \cdot 22} + \dots + \frac{1}{43 \cdot 50} = \frac{4 + 3 + 5 + 7 + \dots + 49}{217}$$

Thực hiện phép tính:

Lời giải

$$\begin{aligned} & \frac{1}{7 \cdot 8} + \frac{1}{8 \cdot 15} + \frac{1}{15 \cdot 22} + \dots + \frac{1}{43 \cdot 50} = \frac{4 + 3 + 5 + 7 + \dots + 49}{217} \\ &= \frac{1}{7} \cdot \frac{1}{8} - \frac{1}{8} + \frac{1}{8} - \frac{1}{15} + \dots + \frac{1}{43} - \frac{1}{50} = \frac{1 + 3 + 5 + 7 + \dots + 49}{217} \\ &= \frac{1}{7} \cdot \frac{1}{50} - \frac{1}{50} = \frac{12 \cdot 50 + 25}{217} \\ &= \frac{1}{7} \cdot \frac{49}{50} = \frac{625}{7 \cdot 50 \cdot 7 \cdot 31} \\ &= \frac{7 \cdot 7 \cdot 2 \cdot 2 \cdot 5 \cdot 31}{7 \cdot 2 \cdot 5 \cdot 5 \cdot 7 \cdot 31} \\ &= \frac{2}{5} \end{aligned}$$

Câu 54. (HSG 7 trường Lý Thường Kiệt 2017 - 2018; trường Giao Tân 2016 - 2017)

$$A = \frac{1}{100} - \frac{1}{100 \cdot 99} - \frac{1}{99 \cdot 98} - \frac{1}{98 \cdot 97} - \dots - \frac{1}{3 \cdot 2} - \frac{1}{2 \cdot 1}$$

Rút gọn

Lời giải

$$\begin{aligned} 1.1) A &= \frac{1}{100} - \frac{1}{100 \cdot 99} - \frac{1}{99 \cdot 98} - \frac{1}{98 \cdot 97} - \dots - \frac{1}{3 \cdot 2} - \frac{1}{2 \cdot 1} \\ A &= \frac{1}{100} - \frac{1}{100 \cdot 99} + \frac{1}{99 \cdot 98} + \frac{1}{98 \cdot 97} + \dots + \frac{1}{3 \cdot 2} + \frac{1}{2 \cdot 1} \\ A &= \frac{1}{100} - \frac{1}{100 \cdot 99} + \frac{1}{99 \cdot 98} + \frac{1}{98 \cdot 97} + \dots + \frac{1}{97 \cdot 98} + \frac{1}{98 \cdot 99} + \frac{1}{99 \cdot 100} \\ A &= \frac{1}{100} - \frac{1}{100} + \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{97} - \frac{1}{98} + \frac{1}{98} - \frac{1}{99} + \frac{1}{99} - \frac{1}{100} \\ A &= \frac{1}{100} - \frac{1}{100} + \frac{1}{100} = \frac{49}{50} \end{aligned}$$

Câu 55. (HSG 7 trường thực hành Sài Gòn 2017 - 2018)

$$N = \frac{1}{1000} - \frac{1}{1000 \cdot 999} - \frac{1}{999 \cdot 998} - \frac{1}{998 \cdot 997} - \dots - \frac{1}{3 \cdot 2} - \frac{1}{2 \cdot 1}$$

Tính nhanh :

Lời giải

$$\begin{aligned}
 N &= \frac{1}{1000} - \frac{1}{1000.999} + \frac{1}{999.998} + \frac{1}{998.997} + \dots + \frac{1}{3.2} + \frac{1}{2.1} \\
 &= \frac{1}{1000} - \frac{1}{1000} + \frac{1}{1000} - \frac{1}{998} + \frac{1}{999} - \frac{1}{997} + \frac{1}{998} + \dots + \frac{1}{2} - \frac{1}{3} + 1 - \frac{1}{2} \\
 &= \frac{1}{1000} - \frac{1}{1000} + \frac{499}{500} = \frac{499}{500}
 \end{aligned}$$

Câu 56. (HSG 7 huyện Thái Thụy 2017 - 2018)

$$P = 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{2012}(1+2+3+\dots+2012)$$

Tính:

Lời giải

$$\begin{aligned}
 P &= 1 + \frac{1}{2}(1+2) + \frac{1}{3}(1+2+3) + \frac{1}{4}(1+2+3+4) + \dots + \frac{1}{2012}(1+2+3+\dots+2012) \\
 &= 1 + \frac{1}{2} \cdot \frac{2.3}{2} + \frac{1}{3} \cdot \frac{3.4}{2} + \frac{1}{4} \cdot \frac{4.5}{2} + \dots + \frac{1}{2012} \cdot \frac{2012.2013}{2} \\
 &= \frac{2}{2} + \frac{3}{2} + \frac{4}{2} + \frac{5}{2} + \dots + \frac{2013}{2} = \frac{1}{2}(2+3+4+\dots+2013) \\
 &= \frac{1}{2}(1+2+3+\dots+2013-1) = \frac{1}{2} \cdot \frac{2012.2013}{2} = \frac{2025077}{2}
 \end{aligned}$$

Câu 57. (HSG 7 huyện Hòa Bình 2016 - 2017)

$$A = \frac{1}{1.2} + \frac{1}{2.3} + \frac{1}{3.4} + \dots + \frac{1}{99.100}$$

Tính giá trị biểu thức:

Lời giải

$$A = \frac{1}{1.2} + \frac{1}{2.3} + \dots + \frac{1}{99.100} = 1 - \frac{1}{2} + \frac{1}{2} - \frac{1}{3} + \dots + \frac{1}{99} - \frac{1}{100} = \frac{99}{100}$$

Câu 58. (HSG 7 huyện Đức Thọ 2015 - 2016)

$$\frac{1}{7.8} + \frac{1}{8.15} + \frac{1}{15.22} + \dots + \frac{1}{43.50} + \frac{4-3-5-7-\dots-49}{217}$$

Thực hiện phép tính:

Lời giải

$$\begin{aligned}
 &\frac{1}{7.8} + \frac{1}{8.15} + \frac{1}{15.22} + \dots + \frac{1}{43.50} + \frac{4-3-5-7-\dots-49}{217} \\
 &= \frac{1}{7.8} - \frac{1}{8} + \frac{1}{8} - \frac{1}{15} + \frac{1}{15} - \frac{1}{22} + \dots + \frac{1}{43} - \frac{1}{50} + \frac{5-(1+3+5+7+\dots+49)}{217} \\
 &= \frac{1}{7.8} - \frac{1}{50} + \frac{5-(12.50+25)}{217} \\
 &= \frac{1}{7} \cdot \frac{49}{50} - \frac{625}{7.31} = \frac{7.7.2.2.5.31}{7.2.5.5.7.31} = -\frac{2}{5}
 \end{aligned}$$

Câu 59. (HSG 7 trường Nghĩa Điền, Tư Nghĩa 2017 - 2018)

Tính tổng: $A = \frac{2}{1.3} + \frac{2}{3.5} + \frac{2}{5.7} + \dots + \frac{2}{2011.2013}$

Lời giải

$$\begin{aligned} A &= \frac{2}{1.3} + \frac{2}{3.5} + \frac{2}{5.7} + \dots + \frac{2}{2011.2013} \\ &= 1 - \frac{1}{3} + \frac{1}{3} - \frac{1}{5} + \frac{1}{5} - \frac{1}{7} + \dots + \frac{1}{2011} - \frac{1}{2013} = 1 - \frac{1}{2013} = \frac{2012}{2013} \end{aligned}$$

Câu 60. (HSG 7 Thi thử Hiệp Hòa 2022 – 2023 lần 2)

Cho $A = \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{1000^2}$. Chứng minh rằng: $A < \frac{25}{36}$.

Lời giải

$$A = \frac{1}{4} + \frac{1}{9} + \dots + \frac{1}{1000^2}$$

$$\text{Đ } A < \frac{1}{4} + \frac{1}{9} + \frac{1}{3.4} + \frac{1}{4.5} + \dots + \frac{1}{999.1000}$$

$$\text{Đ } A < \frac{1}{4} + \frac{1}{9} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \dots + \frac{1}{999} - \frac{1}{1000}$$

$$\text{Đ } A < \frac{1}{4} + \frac{1}{9} + \frac{1}{3} - \frac{1}{1000}$$

$$\text{Đ } A < \frac{25}{36} - \frac{1}{1000} < \frac{25}{36} \text{ (đpcm)}$$

$$\text{Vậy } A < \frac{25}{36}.$$

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